

ECONOMIC-MATHEMATICAL MODEL FOR THE EXAMINATION OF CLUSTERS' SUSTAINABILITY

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ABSTRACT

Economic, political, and social tendencies have brought new actors into the picture. Alliances, business networks, and a special type of cooperation called "cluster" now play a more and more important role in the development of countries' economies and in the enhancement of regions' and local companies' competitiveness. The most efficient of these are clusters, which consist of profit oriented, competitive companies, academic institutions, and civil establishments trying to collaborate with each other toward a common goal and taking advantages of geographical proximity and the coordination of capacities. The most essential condition for establishment of a cluster is that a certain number of enterprises take part in the collaboration. This number can be defined as "critical mass," because it generates processes that attract other institutions to the region to utilize the possibilities of spatial concentration and to take advantage of positive local conditions. The specific quantity of institutions in "critical mass" is related to the financial requirements of the given cluster. Reciprocal access to the organizations' incomes is necessary to long-term operation, and this can be difficult without the right number and combination of members. I created an economic-mathematical model suitable for examination of networks' sustainability, considering financial circumstances and shareholder base.

Keywords: cluster, model, sustainability, financial structure, shareholder base

INTRODUCTION

My paper is about a mathematical-economic model I have created for the examination of the sustainability of business networks as clusters.

Today's economic map of the world is characterized by what Porter calls clusters: critical masses in one place made of linked industries and institutions ranging from suppliers through universities to government agencies that enjoy unusual competitive success in a particular field (Porter, 1998).

Clusters have been explored by several authors dating back to Weber (1929) and Marshall (1920, 1923). More recent reviews include those of Lloyd and Dicken (1977), Krugman (1991) and Enright (1998) but the most quoted explanation was identified by Porter (1998).

According to Porter's theory clusters are geographic concentrations of interconnected companies, specialized suppliers, service providers, and associated institutions in a particular field that are present in a nation or region. Clusters affect competition in three broad ways: first, by increasing the productivity of companies

based in the area; second, by setting the direction and pace of innovation; and third, by stimulating the formation of new businesses within the cluster. Geographic, cultural, and institutional proximity provides companies with special access, closer relationships, better information, powerful incentives, and other advantages that are difficult to tap from a distance (*Porter, 1998*).

The maintenance of these initiations is a popular topic of discussions among the specialists and the policy makers everywhere in the world and that was the reason why I have dealt with the modelling of this problem.

If we examine the countries where cluster policies have been determined and applied we can experience that there is no generally accepted practice which can be used successfully everywhere and could be adopted without difficulties by other countries. This is due to the divided opinions on the maintenance, membership and financing of clusters depending on the national capabilities, special circumstances and economic backgrounds. The participants of collaboration require external support either from the government or the EU especially in the first phase of the cooperation. The network needs a significant amount of investment in order to create a favourable business environment and overcome problems related to the difficulties of communication and the lack of trust. In addition the invested capital does not generate profit and quick return automatically. However policy makers emphasize the importance of clusters' self-sustaining capacity which is essential to the long-term sustainability of networks. This capacity can only be realized in the end of the second or at the beginning of the third phase of the life-cycle. Summarizing the requirements joint funding seems to be the best solution. Experiences highlight the fact that exclusive state aids result in the formation of clusters where no real cooperation exists but which were established only for the acquisition of EU sources. The financial question is strongly associated with the membership composition of these organizations. The number of contributors operating in the same industrial branch, such as enterprises, support and background institutions, has to reach a minimum. This so-called 'critical mass' is necessary to realize the self-financing of clusters and generate positive processes attracting other institutions to take advantage of local externalities and geographical proximity thus enhancing the area's competitiveness.

MATERIALS AND METHODS

More experts have examined the dynamics and the "critical mass" of clusters with mathematical tools who had a very great impact on me and on my research activity. I emphasize among them *Brenner (2004)* and his book titled „Local Industrial Clusters, Existence, Emergence and Evolution” and the book „Clusters, Networks and Innovation” which was edited by *Breschi and Malerba (2006)*. In addition to these the basic idea of my mathematical-economic model was inspired by a biological model examining the sustainability of a community consisting of two homogeneous groups with different functions, tasks and responsibilities. This article was written by three American researchers called *Haque, Egerstedt and Martin (2010)*. The lion model they created dealt with African Lions, *Panthera leo* that live in well-defined

social structures known as prides. Typically, these prides consist of 1-3 adult males and 2-9 adult females along with their dependent cubs. Females are usually in charge of hunting for food, while males are responsible for territorial defense. The American research team determined whether the group is sustainable with a given number of males and females. By sustainable, they meant that the females can hunt sufficient prey to feed the entire pride and at the same time, there is an adequate number of males to patrol the territory.

I applied the same method because I divided the cluster initiation into two homogenous groups consisting of productive and improductive members where both of these teams had their own tasks. I tried to transpose the criteria of the lion model into mine making some adjustments on the way.

CLUSTER-MODEL

Meanings of the abbreviations and notations

The abbreviation C arises from the English expression „cluster” which is a special type of business network. I divided the cluster’s membership into two groups. These consist of J productive and improductive members.

It is self-evident that the members count for different advantages and benefits from the co-operation, that is the reason why they choose to collaborate against the intense competition. I assumed that all of these gains can be realized in money. M is the cluster’s resource demand, m_j stands for the aim for money of productive members, m_i means the expectation for sources of improductive members which are determined for one year.

The measure of annual membership fee which has to be paid into the cluster’s budget by the members is t , where the subscripts refer to the groups which they belong to. Accordingly t_j marks the annual membership fee of productive members while t_i shows the annual membership fee of improductive members.

A special part of incomes within the cluster arises from external services which are provided for outsiders by productive members therefore I marked it with s_j .

Pr is the abbreviation of a bivariate function which describes the probability of applications’ chances of winning. These applications were submitted by groups formed ad hoc within the cluster to apply for money from the European Union and from the Hungarian government.

The parameter g illustrates the number of project teams created within the cluster to submit applications and raise fund. Let me assume that all of them include j_g productive and i_g improductive cluster members.

I would like to emphasize that the same (optimal) productive-improductive proportion can be experienced in all groups of a cluster which have been formed to submit applications and it is valid in the whole network too. I marked this rate with k .

THE CLUSTER MODEL’S ASSUMPTIONS

The membership of the cluster consists of two groups. One of them involves productive members like enterprises, suppliers and service providers while the

other involves unproductive members such as universities, research institutes, consulting companies and other non-profit organizations. This condition can be described with the following equation:

$$C=I+J. \tag{1}$$

The long-term operation and the sustainability of the cluster require a minimum level of annual sources necessary to finance the operating expenses and to facilitate the provision of services for the cluster's members (for example to maintain the common webpage, organize different trainings, professional tours and factory visits, edit and send out the annual newsletter and a lot of advertisements, represent the cluster's members at conferences, workshops and exhibitions, obtain money for the maintenance of the cluster's management organization etc.). Members' aim for money can be evaluated with the following formula:

$$M \min(I, J) = m_I \cdot I + m_J \cdot J. \tag{2}$$

The money flowing into the cluster within a year is utilized jointly by the partaking members of the co-operation to realize their common goals. Therefore when I simulated the model I assumed that the share from the network's money is the same for the productive and for the unproductive members. That was the reason why I substituted the same amount into. This amount can be defined as one member's need who decided to participate in the collaboration to complete its missing capacities. This money should be realized from the annual sources by each organization regardless of its being productive or unproductive. Concerning the amount we can differentiate it between the two groups. If the contribution of the productive members to the cluster's maintenance is more important than that of the unproductive one, we can allocate a bigger amount for the unproductive members which means that $m_J > m_I$.

1. The annual revenue of the cluster can derive from membership fees, from external services and from winning projects which can be realized in one year as the following equation shows:

$$M(I, J) = t_I \cdot I + t_J \cdot J + s_J \cdot J + g \cdot \bar{M} \cdot \Pr(j_g, i_g), \tag{3}$$

where \bar{M} signs the amount of the average annual money acquired by the applying groups formed ad hoc while $\Pr(j_g, i_g)$ shows the probability which describes applications' chance of winning.

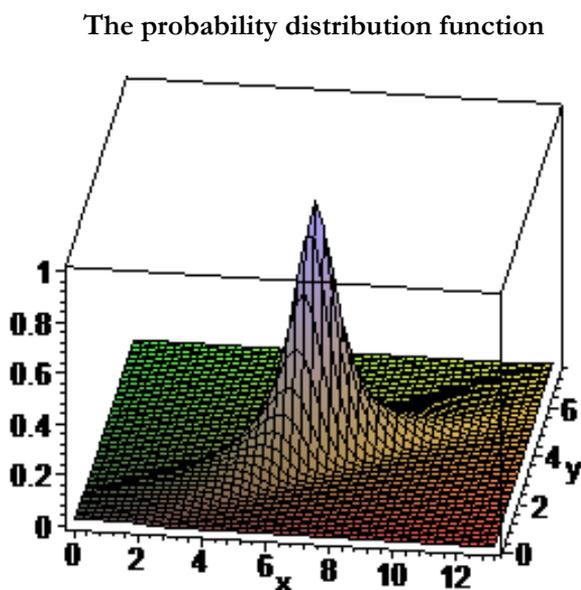
2. $\Pr(j_g, i_g) \in [0, 1]$ gives a probability whose value can move between 0 and 1 and it is described by a bivariate quadratic performance curve. One of the possible functions can be written down by the following equation:

$$\Pr(x, y) = \frac{1}{(x - ky_0)^2 + k^2 \cdot (y - y_0)^2 + e^{(x - ky)^2}}, \tag{4}$$

and another condition is valid at the same time: $x_0 = ky_0$. This curve reflects the 5th hypothesis very well. The applications' chances of winning are higher if the groups formed to write proposals consist of more productive and unproductive members till they reach the upper limit of the membership. After that the coordination and

the harmonization of the members' interests become more difficult and the efficiency decreases. The highest chance of winning can be experienced in the environment of the $x=ky$ line, and $\mathbf{Pr}(x, y)$ function's absolute maximum is in the (x_0, y_0) point. The figure of the curve can be followed up at the first illustration (Figure 1).

Figure 1



3. Each group within the cluster formed to apply for governmental support has to include productive and unproductive members. Let me repeat the former remark that the same (optimal) productive-unproductive proportion can be experienced in all groups and is valid in the whole network too. This rate was marked with k .

I assume that every member of the cluster is allowed to take part in a maximum of one application (and since participation in projects is not obligatory there can be passive members in the cluster). This assumption can be described with the following formulas:

$$g \cdot j_g \leq J, \tag{5}$$

$$g \cdot i_g \leq I, \tag{6}$$

$$k = \frac{J}{I} = \frac{j_g}{i_g}. \tag{7}$$

4. According to my notion of sustainability, a cluster can be maintained in the long term if it's financially feasible without any problems which means that the following condition prevails:

$$J \cdot (m_J - t_J - s_J) + I \cdot (m_I - t_I) \leq g \cdot \bar{M} \cdot \Pr(j_g, i_g). \tag{8}$$

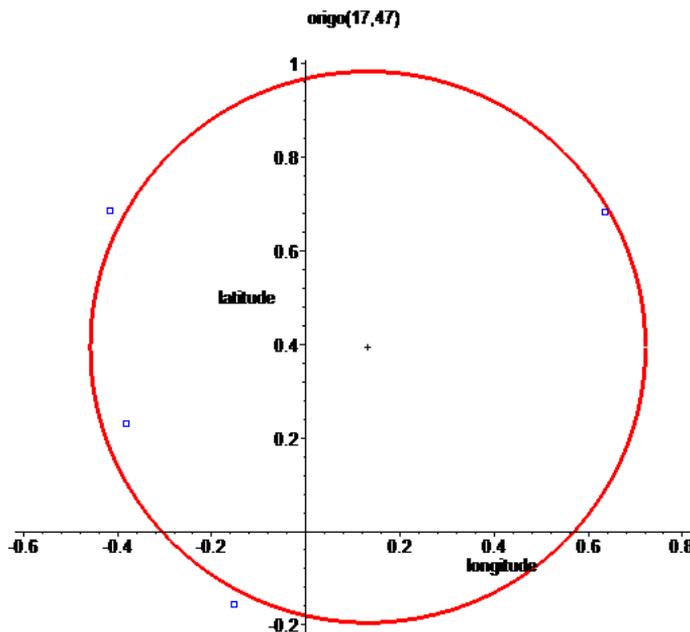
5. My model has an additional component because it is based on the West-Transdanubian Region and I assumed that the membership of the cluster does not cross the imaginary borders. Seven statistical regions were adopted in Hungary by the decision of the Parliament in 1998. The Region of West-Pannonia includes three counties: Győr-Moson-Sopron, Vas and Zala counties. The region borders four countries: Austria in the West, Slovenia and Croatia in the Southwest and Slovakia in the North. I utilized the information about the Region's area because its extension is 11 209km². I experienced that the unproductive members of the clusters such as universities, research institutes, the Regional Development Agency's offices and the largest consulting and tender writer companies concentrate in the 3 counties' centres: Győr, Szombathely and Zalaegerszeg and in a big town called Sopron. I considered 4 cities as unproductive centres where unknown numbers of unproductive organizations can be found facilitating the productive members' more effective operations.

I illustrated the four towns with their geographic coordinates and I fitted a circle around them with the Ordinary Least Squares as you can see in the *Figure 2*. After that the circle which I got was increased to the size of the region's extension, to 11 209km².

The circle which I fitted around the region's 4 big towns (unproductive centres), Sopron, Győr, Szombathely and Zalaegerszeg can be seen in the *Figure 2*.

Figure 2

The circle fitted around Győr, Sopron, Szombathely and Zalaegerszeg (unproductive centres)



SIMULATION AND RESULTS

I tried to provide the computerized testing of the model with actual data from the real economic environment. In this intention I could count on managers directing the clusters' operation in the West-Pannon Region. A chosen cluster supplied the information used in the model to simulate its function.

In the first case I tested the members of the chosen cluster pay an annual fee to facilitate long term operation and financial security. The amount of this fee is $t_j = 100$ for the productive members and $t_i = 50$ for the unproductive ones. (The amounts are given in thousand forints abbreviated HUF.)

The optimal productive-unproductive proportion in the cluster and in all of the groups formed within the cluster was assumed as 2 which means that $k=2$.

The cluster has other incomes too which arise from external services. These are provided for outsiders by productive members. I evaluated the amount of the annual service charge and determined its measure in $s_j = 67$ (thousand HUF).

I assumed that 3 groups formed within the cluster to write applications and to try to get money for the co-operation in the framework of the cluster. It means that $g=3$. I had another assumption for the composition of proposal writer groups too because I assumed that all groups consist of 5 productive and 2 unproductive members: $j_g=5$ és $i_g=2$.

The members share the sources realized in one year equally because they utilize the money jointly. I assumed that the average annual resource demand of unproductive and productive members is the same. This amount is $m_j = m_i = 500$ (thousand HUF).

The question is given: Which clusters are sustainable in long term under these conditions if the network's composition and membership is taken into account?

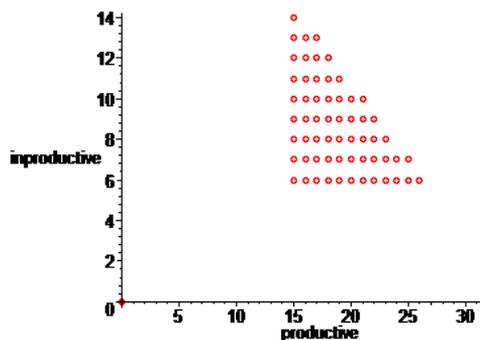
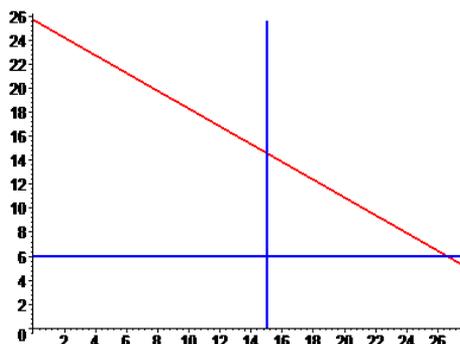
The answer which I got from the simulation is illustrated in the *Figure 3*.

The three cases differ from each other only in one condition, every other parameter is the same. The amount of money acquired by winning tenders is decreased gradually. The result can be followed up in the *Figure 3*.

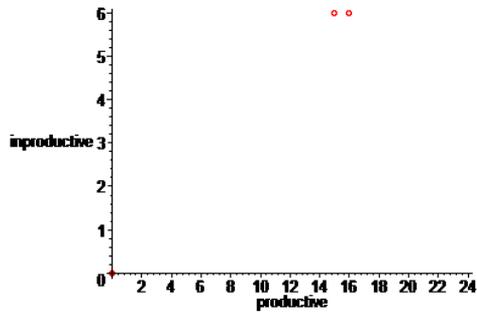
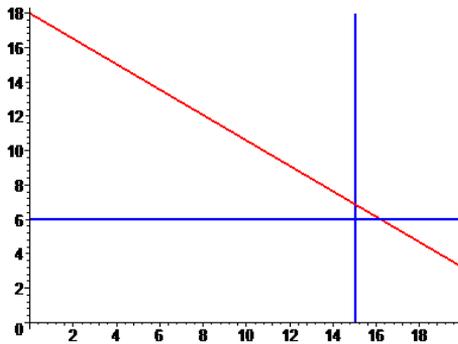
Figure 3

The result of the model's first simulation

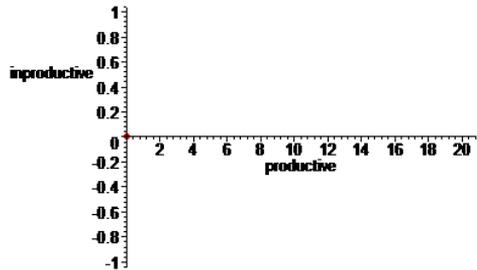
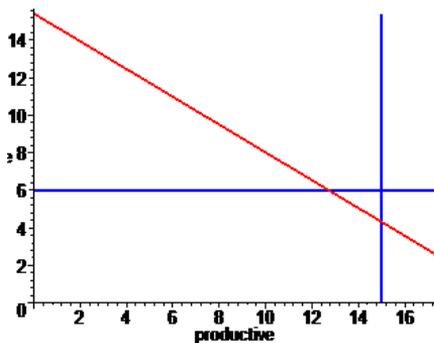
1. case $\bar{M} = 50\,000\,000$ HUF/year



2. case $\bar{M} = 35\,000\,000$ HUF/year



case $\bar{M} = 30\,000\,000$ HUF/year



In the first case I assumed that the groups writing proposals could acquire 50 million HUF as support from the Hungarian government and from the European Union. I examined the conditions which influence the cluster's sustainability and I determined that under the given circumstances 60 different networks of different sizes and shareholder basis are viable. The solution can be represented by a triangle bounded by 3 lines arising from the equations in the 5th and 6th points. When I reduced the external sources I experienced the decrease of the number of sustainable clusters. When the amount of support was 35 million HUF, the line representing the financial criteria changed and therefore the area of the triangle (solution of the mathematical exercise) became smaller and it resulted less clusters that can be maintained in long term. Only 2 clusters can operate under these circumstances. In the third case the assumed external contribution to the cluster's budget was 30 million HUF. When I left all the other conditions unchanged I experienced that there was no co-operation in the framework of clusters that could be maintained in long term.

During the first simulation I became curious wheatear the number of the sustainable cluster initiations will change and the chance of survival will be higher or not if I increase both the amounts of annual fees for the productive and inproductive members too at the same time when the external sources are reduced. This situation means that the contribution of the collaboration's members lifts while the proportion of the supports which clusters get by applications from the

government decreases within the financial structure of the cluster. I simulated this case and I represented the result on the *Figure 4*.

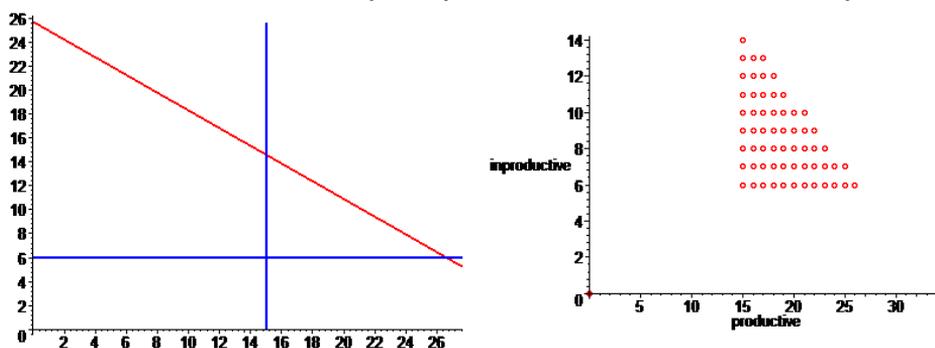
I substituted the same data I had used in the first simulation before I started to decrease the external sources of the cluster but in the second testing I reduced the governmental support and increased the annual fees of productive and improductive members parallel. This solution is promoted by most of the experts dealing with the financial maintenance of clusters because they emphasize the importance of the organizations' self-preservation which is essential to the long-term operation. I assumed the same measurement (20 percent) in the reduction of the governmental support and in the increase which happened in the amounts of the annual fees. It means that 3 parameters changed parallel in the model: \bar{M} which signes the amount of the average annual money acquired by the winning applications decreases by 20 percent year by year while t_j and t_i (the annual membership fees of the productive and improductive members) increase by 20 percent every year. Every other data is unchanged compared to the initial state.

The result of the simulation can be studied in the *Figure 4*.

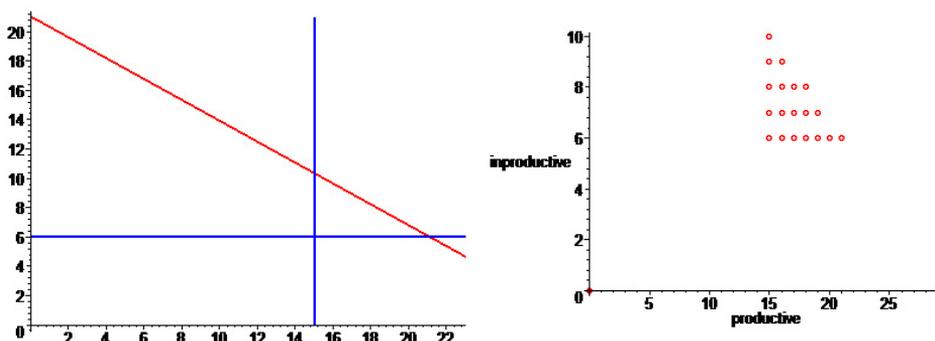
Figure 4

The result of the model's second simulation

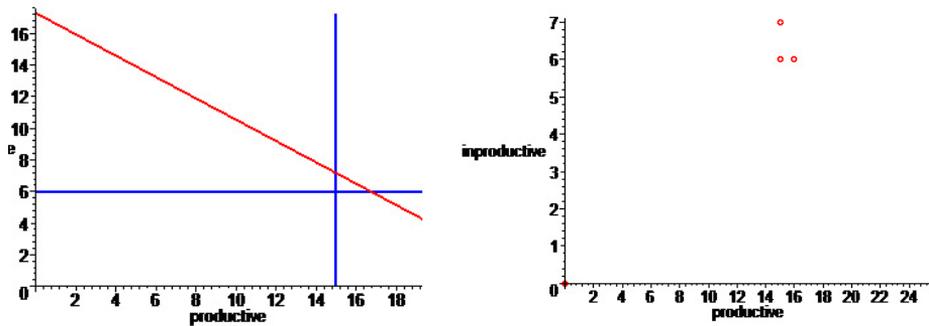
1. case $\bar{M} = 50\,000\,000$ HUF/year, $t_j = 100\,000$ and $t_i = 50\,000$ HUF/year



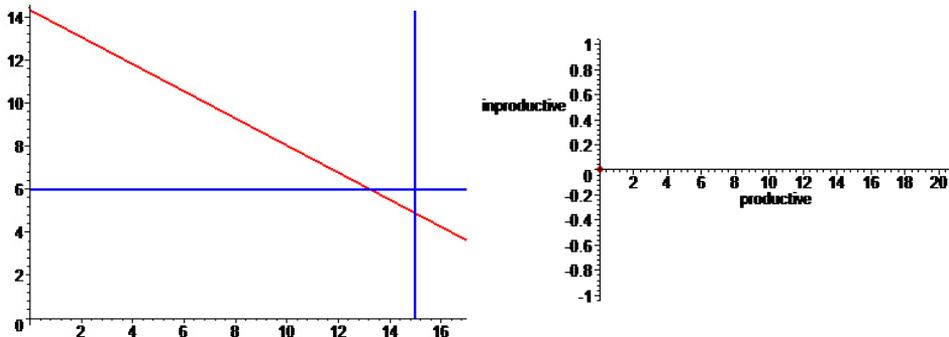
2. case $\bar{M} = 40\,000\,000$ HUF/year (-20 percent), $t_j = 120\,000$ and $t_i = 60\,000$ HUF/year (+20 percent)



3. case $\bar{M} = 32\,000\,000$ HUF/year (-20 percent), $t_j = 144\,000$ and $t_i = 72\,000$ HUF/year (+20 percent)



4. case $\bar{M} = 25\,600\,000$ HUF/year (-20 percent), $t_j = 172\,800$ and $t_i = 86\,400$ HUF/year (+20 percent)



In the first case when the data were the same as in the initial state of the first simulation the result was the same because 60 different networks with different sizes and shareholder basis were viable. One year later the governmental support decreased by 20 percent and the external sources represented such a big proportion in the financial structure of the cluster that the reduction of them could not be compensated by the increase of the annual membership fees (which is a typical characteristic of the Hungarian cluster initiations) the number of the sustainable clusters fell down and only 19 clusters stayed in the examination which have a good chance to the long-term operation among the given circumstances. I repeated the same changes in the following two years too but the number of the sustainable clusters decreased year by year. The conclusion of the second simulation is that without a balanced financial structure - where the external sources do not represent an extremely big proportion in the financial structure like in Hungary - it can not be realized that the decreasing governmental sources can be substituted by the increase of the participants' contribution and by the growing membership fees. Cluster initiations in Hungary depend on external sources too much therefore they need to be promoted by the government especially in the first some years of their operation.

CONCLUSIONS

My goal was to create a simple mathematical-economic model which is suitable to examine clusters' and other business networks' chance of survival under different circumstances. I wanted to take the financial conditions into account just as much as the shareholder base. I am aware of the fact that my model has a lot of deficiencies because of the simplifications and it needs to be improved, and I am planning to develop it in the future.

However it can be determined from this model that clusters need to collect membership fees in addition to gathering external sources and support from the government. The consequence that the size of networks cannot be increased flippantly can also be instructive. If the sources acquired from applications decrease they can be substituted with the increase of membership fees and service charges which can jeopardize the co-operation and the survival of the cluster should its members refuse payment.

REFERENCES

- Brenner, T. (2004): *Local Industrial Clusters: Existence, Emergence and Evolution*. Routledge Studies in Global Competition. Hardcover
- Breschi, S., Malerba, F. (eds.) (2006): *Clusters, Networks and Innovation*. Oxford : University Press
- Enright, M. J. (1998): *Regional Clusters and Firm Strategy*. In: A.D. Chandler Jr., P. Hagström and Ö. Sölvell (eds.): *The Dynamic Firm*, Oxford University Press : Oxford, 315–342. p.
- Haque, M., Egerstedt, M., Martin, C. (2010): *Sustainable Group Sizes for Multi-Agent Search-and-Patrol Teams*. [online] <URL: http://users.ece.gatech.edu/~magnus/Papers/lion_MTNS2010.pdf> [10-01-2011]
- Krugman, P. (1991): *Increasing Returns and Economic Geography*. In: *Journal of Political Economy*, 99. 483-499. p.
- Lloyd, P., Dicken P. (1977): *Location in Space: A theoretical approach to economic geography*. 2nd Edition. Harper & Row : London
- Marshall, A. (1920): *Principles of Economics*. 8th Edition. Mcmillan and Co. : London
- Marshall, A. (1923): *Industry and Trade*. 4th Edition, Macmillan and Co. : London
- Porter, M.E. (1998): *Clusters and the New Economics of Competition*. *Harvard Business Review*, 77-90. p.
- Weber, A. (1909, 1929 reprint): *Theory of the Location of Industries*. The University of Chicago Press : Chicago and London