A METHODOLOGICAL STUDY ON LOCAL APPLICATION OF THE FAO-56 PENMAN-MONTEITH REFERENCE EVAPOTRANSPIRATION EQUATION

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Abstract

In our study we discuss the theory and application of the FAO-56 Penman-Monteith equation to count reference evapotranspiration (ET_0). Establishing ET_0 derived from meteorological variables, together with crop coefficient (K_c), the actual ET could be estimated easily for a desired time-period (from hours to seasons). Daily evapotranspiration of common reed (*Pragmites australis*) was measured in evapotranspirometers. The measured ET was used to calculate K_c locally. Our study confirmed the earlier investigations that reed stand transpired more water than open water bodies in hot season.

Key-words: actual evapotranspiration, reference evapotranspiration, crop coefficient

Összefoglalás

Megfigyelésünkben a FAO-56 módszer néven ismert Penman-Monteith egyenletet és annak egy lehetséges gyakorlati alkalmazását mutatjuk be hazai példán. A dolgozat célja a FAO-56 módszer helybeli alkalmazhatóságának áttekintése, valamint egy gyakorlati példa alapján történő felhasználása volt. A referencia evapotranspiráció és a növénykonstans segítségével meghatározhatjuk egy adott növény faj tényleges párolgását különböző időintervallumokra. A napi nád párolgását evapotranszspirométerben mértük, melyből a nád növénykonstansait származtattuk. Bár megfigyelésünkben a cél nem a konkrét ETc meghatározás volt, a mintapéldánk alapján 2007-ben a nád párolgása mintegy 30%-kal meghaladta a szabad vízfelület evaporációját, mely összhangban van a korábbi irodalmi adatokkal.

Introduction

Evaporation and transpiration are important components of water balance because approximately 60% of precipitation is returned to the atmosphere by evaporation and transpiration in Europe (Baumgarter and Reichel, 1975). These two compounds take also dominant part in determination of the soil moisture; and both of them are major factors in crop water supply analysis and irrigation design.

Actual evapotranspiration (ET_c) is the combination of soil evaporation and crop transpiration under actual meteorological (environmental) conditions. The key components of evapotranspiration processes are: meteorological components (radiation, temperatures, vapour pressure, wind speed), soil-moisture components (available soil water), and plant factors (species, stage of growth, plant density, plant height, leaf area index - LAI etc.).

In water balance, the size of actual evapotranspiration (ET_c) over the long term is more difficult to estimate than either precipitation or streamflow, so we have to find the empirical solution to estimate ET_c.

Allen et al. (1998) gave the basic definition of ET_0 as follows:

"The evapotranspiration rate from a reference surface, not short of water, is called the reference crop evapotranspiration or reference evapotranspiration and is denoted as ET_0. The

reference surface is a hypothetical grass reference crop with specific characteristics". The only factors affecting ET_0 are climatic parameters. Penman model is widely used in ET estimation because of its sound analytical basis (Sun and Song, 2008).

Knowing ET_o, ET_c can be determined easily using a specific coefficient named crop coefficient (K_c). Great number of studies had been dealing with estimating wetland evapotranspiration by K_c application where reed was included (Headley et al. 2012, Borin et al. 2011, Herbst and Kappen 1999, Hargreaves 1994). Published results differed greatly irrespective to climatic conditions and used methodology.

The purpose of the study was to examine the applicability of the Penman-Monteith Method with locally measured data. As an example we estimated local ET_c from crop coefficients and reference crop evapotranspiration at a daily timestep using standard meteorological data only. According to one seasons sample data for reed evapotranspiration or crop coefficients, the analysis of reed ET_c or K_c were excluded from this work.

Material and Methods

Theory of the Applied Method: Evaluation of Reference Crop Evapotranspiration, ET_0 (FAO-56 Method)

Many empirical methods have been developed over the last five decades estimating evapotranspiration from climatic variables. Some of them derived from the original Penman equation (Penman, 1948) to determine evaporation from open water, bare soil and grass based on a combination of an energy balance and an aerodynamic formula:

$$\lambda E = \frac{[\Delta(R_n - G)] + (\gamma \lambda E_a)}{(\Delta + \lambda)} \qquad (1)$$

where λE is the evaporative latent heat flux [MJ m^{-2} d^{-1}], Δ is the slope of the saturated vapour pressure curve [kPa] R_n is the net radiation flux [MJ m^{-2} d^{-1}], G is the sensible heat flux into the soil [MJ m^{-2} d^{-1}], γ is the psychrometric constant [kPa°C^{-1}], and E_a is the vapour transport of flux [mm d^{-1}].

After various derivation of the original Penman equation the so called Penman-Monteith equation was developed including a bulk surface resistance term by Monteith (1965) :

$$\lambda ET = \frac{\Delta(R_n - G) + \frac{\rho_a C_p (e_s - e_a)}{r_a}}{\Delta + \gamma(1 + \frac{r_s}{r_a})} \qquad (2)$$

where ρ_a is air density [kg m^{-3}], C_p is specific heat of dry air, e_s is mean saturated vapour pressure [kPa], e_a is mean daily ambient vapour pressure [kPa], r_s and r_a are the bulk surface and aerodynamic resistances [s m^{-1}].

The aerodynamic resistance (r_a) determines the transfer of heat and water vapour from the evaporating surface into the air above the canopy (Allen et al. 1998):

$$r_a = \frac{\ln\left[\frac{z_m - d}{z_{om}}\right]\ln\left[\frac{z_h - d}{z_{oh}}\right]}{k^2 u_z} \qquad (3)$$

where z_m is the height of wind measurements [m], z_h is height of humidity measurements [m], d is the zero plane displacement height [m], z_{om} is roughness length governing momentum transfer [m], z_{oh} is the roughness length governing transfer of heat and vapour [m], k is the von Karman's constant: 0.41, u_z is the wind speed at height z [m s^{-1}]

The bulk surface resistance (r_s) describes the resistance of vapour flow through the transpiring crop and evaporating soil surface (Allen et al., 1998):

$$r_s = \frac{r_l}{LAI_{active}} \qquad (4)$$

where r_l is the bulk stomatal resistance of the well-illuminated leaf [s m^{-1}], LAI_{active} is the active sunlit leaf area index [$m^2 m^{-2}$], and $LAI_{active}=0.5LAI$.

An updated and simplified equation of the Penman-Monteith equation was recommended by FAO Expert Consultation on Revision of FAO Methodologies for Crop Water Requirements (Allen et al., 1998) with the FAO-56 Penman-Monteith Equation. This equation utilizes some assumed constant parameters for a reference crop (alfalfa or short cut grass). The adopting characteristics for the reference crop was a hypothetical reference crop with a height of 0.12 m, a surface resistance of 70 s m^{-1} and an albedo value of 0.23 (Allen et al., 1998).

ET_0 parameters measured at different locations or in different seasons are comparable as they refer to the evapotranspiration from the same reference surface, so the only factors affecting ET_0 are climatic parameters.

The basic form of FAO-56 Penman-Monteith ET_0 [mm day^{-1}] Equation (Allen et al., 1998) is as follows:

$$ET_0 = \frac{0.408\,\Delta(R_n - G) + \gamma \frac{900}{T + 273} u_2 (e_s - e_a)}{\Delta + \gamma\,\Delta(1 + 0.34\ u_2)} \qquad (5)$$

where R_n is net radiation at the crop surface [MJ m^{-2} day^{-1}], G is soil heat flux density [MJ m^{-2} day^{-1}], T is mean daily air temperature at 2 m height [°C], u_2 is wind speed at 2 m height [m s^{-1}], e_s is saturation vapour pressure [kPa], e_a is actual vapour pressure [kPa], (e_s-e_a) is saturation vapour pressure deficit [kPa], Δ is slope vapour pressure curve [kPa°C^{-1}], γ is psychrometric constant [kPa°C^{-1}], 0.408 is a conversion factor from MJ m^{-2} day^{-1} to equivalent evaporation in mm/day.

As soil heat flux is relatively small compared to R_n beneath the grass reference surface, and hourly or longer calculation time is used, the G is assumed to be zero.

The slope of the relationship between saturation vapour pressure and temperature**,** Δ [kPa°C^{-1}], is counted by:

$$\Delta = \frac{4098\left[0.6108\ \exp\left(\frac{17.27\ T}{T + 237.3}\right)\right]}{(T + 237.3)^2} \qquad (6)$$

The psychrometric constant**, γ** [kPa °C^{-1}], is given by:

$$\gamma = \frac{c_p P}{\varepsilon\lambda} = 0.664742\ * 10^{-3} P \qquad (7)$$

where P is atmospheric pressure [kPa], λ is latent heat of vaporization, 2.45 [MJ kg^{-1}], c_p is specific heat at constant pressure, $1.013*10^{-3}$ [MJ kg^{-1}°C^{-1}], ε is ratio molecular weight of water vapour/dry air = 0.622.

The atmospheric pressure, P [kPa], is also needed to calculate for Eq. 7:

$$P = 101.3\left(\frac{293 - 0.0065\ z}{293}\right)^{5.26} \qquad (8)$$

where z is elevation above sea level [m], it is 124 m at Keszthely.

The wind speed has to be adjusted to standard height, u_2, [m s^{-1}] of 2 m:

$$u_2 = u_z \frac{4.87}{\ln(67.8\, z_m - 5.42)} \qquad (9)$$

where u_z is measured wind speed at 10.5 m above ground surface [m s^{-1}], z_m is height of measurement above ground surface (10.5 m).

The net radiation (R_n) is the difference between the incoming net shortwave (R_{ns}) and the outgoing net longwave radiation (R_{nl}):

$$R_n = R_{ns} - R_{nl} \qquad (10)$$

The R_{ns} net solar or shortwave radiation [MJ m^{-2} day^{-1}] is given by:

$$R_{ns} = (1-\alpha) R_S \qquad (11)$$

where α is albedo for the reference crop. The R_s incoming solar radiation [MJ m^{-2} day^{-1}] was measured locally by CM-3 pyranometer. The fixed value of 0.23 was used for the albedo.

Net longwave (outgoing) radiation, R_{nl} [MJ m^{-2} day^{-1}] was calculated as follows:

$$R_{nl} = \sigma \left[T_{mean,K}{}^4 \right] (0.34 - 0.14\sqrt{e_a})\,(1.35 \frac{R_S}{R_{SO}} - 0.35) \qquad (12)$$

where σ is Stefan-Boltzmann constant [4.903 10^{-9} MJ K^{-4} m^{-2} day^{-1}], $T_{mean,K}$ is mean temperature during the 24-hour period [K], e_a is actual vapour pressure [kPa], R_s/R_{so} is relative shortwave radiation (limited to ≤ 1.0), R_s is measured solar radiation [MJ m^{-2} day-1], R_{so} is calculated clear-sky radiation [MJ m^{-2} day^{-1}], see Eq. 13.

To get clear-sky solar radiation R_{so} [MJ m^{-2} day^{-1}] the station elevation is required:

$$R_{SO} = (0.75 + 2 * 10^{-5} z) R_a \qquad (13)$$

where R_a is extraterrestrial radiation [MJ m^{-2} day^{-1}]. See Eq. 14.

The extraterrestrial radiation R_a, [MJ m^{-2} day^{-1}] was calculated by:

$$R_a = \frac{24\,(60)}{\pi} G_{SC}\, d_r \left[\omega_S \sin(\varphi)\sin(\delta) + \cos(\varphi)\cos(\delta)\sin(\omega_S)\right] \quad (14)$$

where G_{sc} is solar constant = 0.0820 MJ m^{-2} min^{-1}, d_r is inverse relative distance Earth-Sun (Equation 11), δ is solar declination (Equation 12) [rad], ω_s is sunset hour angle (Equation 17) [rad], φ is latitude [rad]; at Keszthely=0.81.

The lacking Eq. of 11-13 are as follows:

$$d_r = 1 + 0.033\cos\left(\frac{2\pi}{365}J\right) \quad (15)$$

$$\delta = 0.409\sin\left(\frac{2\pi}{365}J - 1.39\right) \quad (16)$$

where J is the number of the day in the year between 1 (1 January) and 365 or 366 (31

December).

The sunset hour angle**,** ωs, is given by:

$$\omega_s = \arccos\left[-\tan(\varphi)\tan(\delta)\right] \quad (17)$$

The vapour pressure deficit, e_s-e_a [kPa] is the difference between the saturation (e_s) and actual vapour pressure (e_a). The saturation vapour pressure, e_s [kPa] is:

$$e_s = 0.6108\exp\left[\frac{17.27\,T}{T + 237.3}\right] \quad (18)$$

The actual vapour pressure, e_a [kPa] is**:**

$$e_a = e_s \frac{RH_{mean}}{100} \quad (19)$$

where RH is relative humidity [%].

The calculation of the crop coefficient as a dimensionless indicator using reference evapotranspiration (Eq. 5) and the measured actual evapotranspiration:

$$K_c = \frac{ET_c}{ET_o} \qquad (20)$$

The site of observation was reed dominated Kis-Balaton wetland. In 2004 reed rhizomes were collected from the Kis-Balaton area and were transplanted into metal containers of Thornthwaite-Matter-type compensation evapotranspirometers of the Meteorological Research Station at Keszthely. The volume of tanks were 4 m^3 (2 × 2 m surface area and 1 m depth) with four replications and were provided unlimited water supply. Measured and reference evapotranspiration (ET_m and ET_o) of common reed was detected between 2005 and 2011, except of 2006.

Meteorological variables were observed at Keszthely Meteorological Research Station (latitude: 46°44′, altitude: 17°14′, elevation: 124 meters above sea level) by a QLC-50 (Vaisala, Helsinki, Finland) automatic climatic station equipped with a CM-3 pyranometer (Kipp & Zonen Corp., Delft, The Netherlands). The combined sensors of air temperature and humidity were placed at standard height 2 meters above the soil surface. Signals from air temperature, humidity, wind speed and radiation were collected every 2 sec, and 10-min means were logged by the QLC-50 station. The height of the anemometer was 10.5 meters.

Practical sample for the applicability of FAO-56 equation (Discussion)

Common reed (*Phragmites australis*) is the most widely distributed wetland crop species in Hungarian areas. Reed covered areas on lakeshore of Balaton exceed 1000 ha (Virág, 1998), and on the Kis-Balaton Water Protection System there are more than 2000 ha (Pomogyi, 2001).

Our ET results will be presented by using 2007 as a sample season because in most months, the seasonal air temperatures and rainfall sums were close to climatic norms. There were two exceptional hot months, the June and July, where monthly air temperatures were about 1°C higher than that of the long-term averages. This hot summer months increased the reed ET significantly.

Figure 1 shows the daily pattern of measured and reference ETs of 2007. These two types of ET were close to each other at the beginning and in the end of the season. In the middle of the season, values of daily ET_m increased on higher extent in reed stand grown in lysimeters than calculated reference ET data. This is in accordance to measurements of earlier investigations for wetland crops at unlimited water supply (Irmak et al., 2013). During 2007, reference and measured seasonal ET totalled 639.8 and 785.5 mm, respectively.

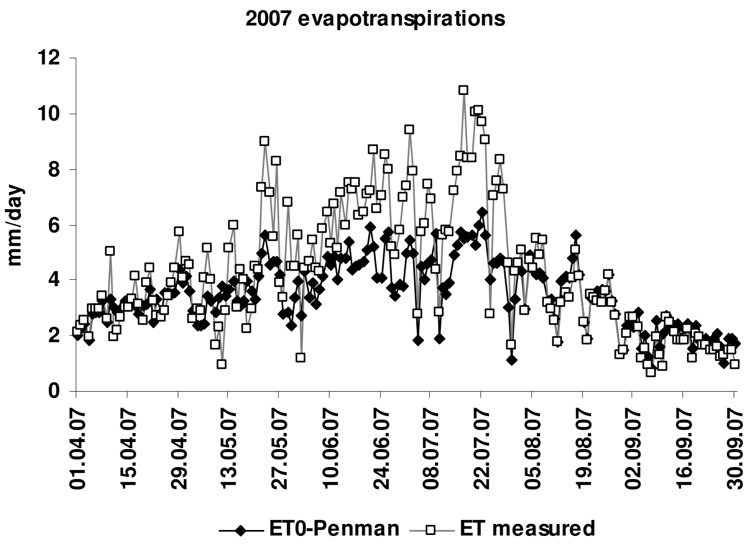

Figure 1. Daily measured and reference evapotranspiration at Keszthely, in 2007

Close relationship was found between measured and ET_o calculated based on meteorological data (Table 1, Fig. 2). Correlation of ET_m and ET_0 was good (R^2= 0.76) and the slope of forcing line through the origin was also close to 1. The values of daily ET_m were on average 27.5% higher than calculated ET_0 for common reed for the whole season of 2007. Relative divergence between the two ETs varied; greatest differences occurred during hot days, while better accordance between ET_m and ET_0 was observed on cooler periods. The RMSD was 1.277 mm/day. This error exceeded with 0.29 mm/day the RMDS for reed ET determined earlier between 2005 and 2011 time-period (Anda et al., 2014). Penman-Monteith equation underestimated reed ET in the Kis-Balaton wetland that could be improved using locally measured K_c values.

Table 1. Table of regression analysis

Regression statistics	
r	0.873
R^2	0.763
corr. R^2	0.762
SE	1.130
n	183

ANOVA

	df	***SS***	***MS***	***F***	***F sig.***
Treatments	1	744.303	744.303	582.477	0.000
Error	181	231.286	1.277		
Total	182	975.589			

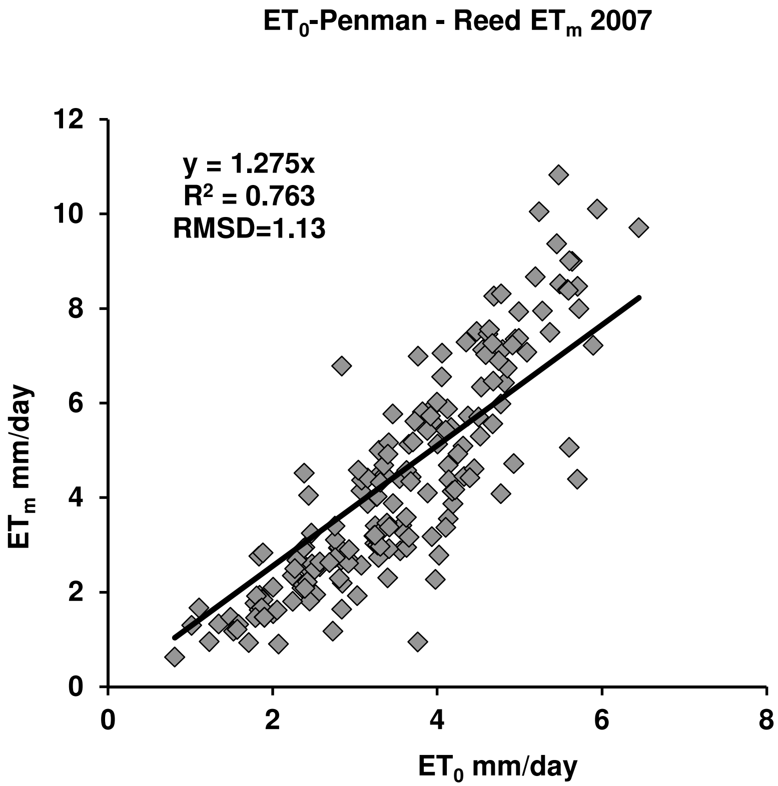

Figure 2. Comparison of measured and reference evapotranspiration in common reed during the growing season of 2007. Reference evapotranspiration was calculated using the Penman-Monteith equation (FAO-56 method).

Ratio of the two ET values provides $K_{c.}$ This coefficient may be used later to calculate actual reed ET based on meteorological parameters only. The monthly mean K_c for 2007 ranged from 0.77 (September) to 1.51 (July) with an average of 1.16. Monthly K_c exhibited an increasing trend until July then K_c means dropped until the end of September. K_c above 1 means that reed stand ET exceeds evaporation of open water bodies (Table 2).

Table 2. Average monthly values of reed K_c

Kc	**April**	**May**	**June**	**July**	**Aug.**	**Sept.**	**Seasonal Avg.**
Monthly Avg.	1.03	1.23	1.4	1.51	0.99	0.77	1.16

Conclusion

The Penman-Monteith equation is widely applied to calculate ET_0 as an international standard introduced by FAO. In our experimental sample, daily evapotranspiration for common reed (*Phragmites australis*) was measured and calculated during the "average" season of 2007. Our study confirmed the earlier investigations that reed stand transpired more water than open water bodies. Later on, locally established reed K_c allows estimation of reed ET based on meteorological parameters only. The Penman-Monteith method suited well in our sample calculation of reed ET_c at Kis-Balaton wetland.

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