



## Comparison of feedback and feed-forward control strategies on a water heater

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### ABSTRACT

*In this work the control of a water-heating equipment with different control strategies was examined. The water flows through the device, while an electric heater raises its temperature. The outlet temperature is controlled by using the power of the heater as the manipulated variable. Measured disturbances are the flow rate and the inlet temperature. The aim of this work is to compare the abilities of control strategies based on feedback and on feed-forward. Two control strategies were tested, one based on a first principle model, and focusing on feed forward, the other based on black box modeling and focusing on feedback. The first approach was basically a constrained inverse controller, but embedded in an IMC structure, to compensate model errors. The second approach was a programmed adaptation with constrained PI controllers. We found that the feed-forward based controllers significantly outperformed the ones based on feedback. The controller performance was much better in simulation experiments, due to the absence of model error.*

(Keywords: IMC, constrained inverse, feed-forward, feedback)

### ÖSSZEFOGLALÁS

#### Előre- és visszacsatoló szabályozási struktúrák összehasonlítása egy vízmelegítő berendezésen

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*Munkánk során egy vízmelegítő berendezés különböző stratégiákkal történő szabályozását vizsgáltuk. Az víz átfolyik az eszközön, miközben egy elektromos fűtőszál fölmelegíti. A cél a kimenő hőmérséklet kézben tartása, a fűtőszál teljesítményével, mint beavatkozóval. Mért zavarásként érik a rendszert a térfogatáram és a belépő hőmérséklet változásai. A cél a visszacsatoláson és az előreccsatoláson alapuló szabályozási struktúrák összehasonlítása volt. Két szabályozási stratégiát hasonlítottunk össze. Az egyik a priori modellen alapult, és az előreccsatolás volt a hangsúlyos eleme, míg a másik fekete-doboz modellen alapult és a visszacsatoláson volt a hangsúly. Az első megoldás egy korlátos inverz IMC struktúrába ágyazva, hogy a modell-hibát kompenzáljuk. A második megoldásban korlátos PI szabályozót használtunk programozott adaptáció mellett. Úgy találtuk, hogy az előreccsatoláson alapuló szabályozó jelentősen jobb szabályozási minőséget biztosít. Szimulációs vizsgálatok során sokkal jobb volt a szabályozás minősége, mivel nem volt modell-hiba.*

(Kulcsszavak: IMC, korlátos inverz, előreccsatolás, visszacsatolás)

## INTRODUCTION

In practical applications feedback control is still the most widespread approach. In the early years of automatic control the controller device was a major constraint on the applicable control strategies, but with the rise of computer process control these limitations vanished and new horizons opened for control strategies. Anyway, the experience gained with analog controllers resulted in practice that new devices with old strategies were applied. To exploit the advantages of computer process control, new controller structures and strategies need to be investigated. Yet the other main approach, the feed-forward is still unfamiliar for some control experts, although it has many benefits to exploit.

In this work the control of a water heating equipment is studied. This equipment can be similar to solar collectors, whose control is summarized well in *Camacho et al.* (2007a; 2007b). The modern controllers are usually model based, and the greatest challenge for this object is modeling. It is a distributed parameter system (DPS), described with a partial differential equation. Another issue is nonlinearity. In recent applications the model error is compensated by adaptation, more detailed and new types of models, including fuzzy models. The nonlinearity is compensated by gain scheduling, predictive control and nonlinear compensators. The control problem is also formulated as an optimal control problem, when associated with predictive control.

In the review of *Padhi and Ali* (2009) the control of DPS has been studied. A very important question in this field is the way we can reduce the partial differential equations to a system of ordinary differential equations. The first solution that comes to mind is discretization of space coordinates, but this can be inefficient or computationally intensive. Other ways usually eliminate the spatial coordinates by integration, Laplace or Fourier transformation.

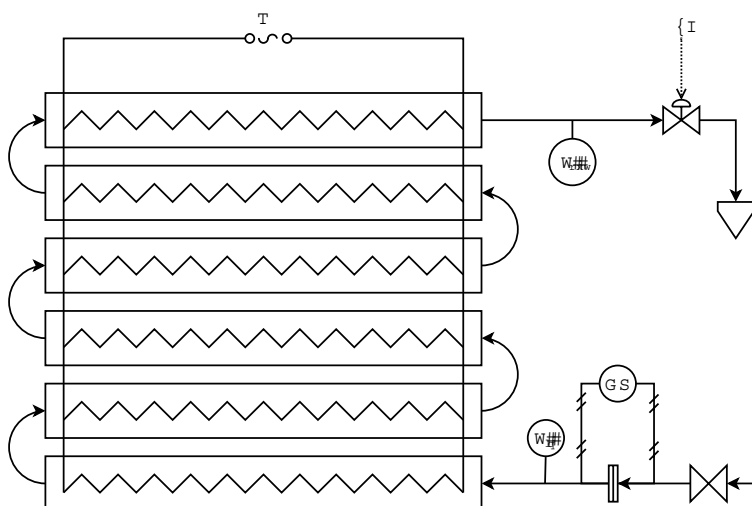
In this article the following section is about the measurement device. The next section introduces modeling and controller synthesis, on basis of first principle models and then based on black box models. The fourth section shows our results in simulation and in physical experiments. The final section summarizes the results and draws conclusions.

### **The measurement equipment**

The main part of the measurement equipment is a tube in which water flows through, and an electric heater raises its temperature (*Figure 1*). The inlet and outlet temperatures are measured by Pt-100 thermometers. The flow rate is measured by an orifice plate, placed on the inlet of the heater tube. The differential pressure is the signal directly obtained from this measurement, but the intelligent measurement device applies digital transformation to the signal. The manipulators on the equipment are the electric power of the heater and a valve for setting flow rate. The power of the heater is driven by a pulse width modulation (PWM) signal. The pneumatic control valve has a slight hysteresis.

The signals, apart from the PWM signal of the electric heater are transferred via Fieldbus H1. The sampling time is 1 second. The interface of the physical system is the Honeywell Experion software. The main control strategy is implemented in Matlab and Simulink that communicates through an OPC server.

The aim of this work is to keep the outlet temperature on a reference signal (controlled variable) by using the power of the heater as manipulator. The flow rate and inlet temperature are measured disturbances. During our measurements the inlet temperature varied only due to disturbances in the pipes of the water supply.

**Figure 1****The schematic representation of the measurement equipment.***1. ábra: A mérőeszköz sematikus rajza.***Modeling and controller synthesis**

Two approaches are common in modeling: first principle and black box modeling. In the following subsections these two approaches and the related controller synthesis are introduced.

**First principle modeling**

First we started our work with building the first-principle model of the process. Only the heat transfer in the liquid phase was analyzed, as our goal is to control the outlet temperature of the liquid flow. Heat transfer towards environment via the pipe wall was neglected, and we only took into account the heat source of the electric heater. It has been assumed that the heat transfer is fast, and thus the whole power of the heater is turned into heat in the liquid flow, no power is retained to heat up the body of the heater itself, or the walls of the equipment. Perfect plug flow of the liquid without axial mixing has been assumed. Perfect mixing of the fluid has been assumed in radial direction, thus the geometric space could be reduced to one dimension, the length. Temperature dependence of the material properties (density and specific heat) have been neglected. The pipe is of the same diameter on its whole length, except some corners, which have been neglected in our model. Following these assumptions the resulting model for an inner point of the system:

$$\frac{\partial T}{\partial t} = -\frac{F}{A} \frac{\partial T}{\partial x} + \frac{Q}{V\rho c_p} \quad (1)$$

Where  $T$  is the temperature of the liquid ( $^{\circ}\text{C}$ ),  $F$  is the volumetric flow rate ( $\text{m}^3/\text{s}$ ),  $A$  is the area available for the flow ( $\text{m}^2$ ),  $Q$  is the power of the heater ( $\text{W}$ ),  $V$  is the volume of the equipment between the two thermometers ( $\text{m}^3$ ),  $\rho$  is the density of the liquid ( $\text{kg}/\text{m}^3$ ),  $c_p$  is the specific heat capacity of the liquid ( $\text{J}/(\text{kg}^{\circ}\text{C})$ ),  $t$  is the time ( $\text{s}$ ),  $x$  is the length coordinate ( $\text{m}$ ).

The initial state and the boundary condition are (starting from steady state):

$$T(0, x) = T_{in}(0) + \frac{Q}{V\rho c_p} \frac{Ax}{F} \quad (2)$$

$$T(t, 0) = T_{in}(t) \quad (3)$$

Where  $T_{in}(t)$  is the inlet temperature ( $^{\circ}\text{C}$ ), function of time.

This is a distributed parameter system, for which the balance equation is a partial difference equation. For control purposes this is not a convenient form, as mostly concentrated parameter models are used in control applications. An easy way to remove the length coordinate is to integrate the (1) equation along the whole length of the tube:

$$\int_0^L \frac{\partial T}{\partial t} dx = \int_0^L \left( -\frac{F}{A} \frac{\partial T}{\partial x} \right) dx + \int_0^L \frac{Q}{V\rho c_p} dx \quad (4)$$

With the introduction an average variable we get an ordinary differential equation:

$$\frac{d\theta}{dt} = \frac{F}{V}(T_{in} - T_{out}) + \frac{Q}{V\rho c_p} \quad (5)$$

Where we used  $\theta$ , an average value for the temperature. The definition and its estimation are as follows:

$$\theta = \frac{1}{L} \int_0^L T dx \approx \frac{T_{in} + T_{out}}{2} \quad (6)$$

Substituting the estimation of  $\theta$  we get the following from equation (5):

$$\frac{dT_{out}}{dt} = \frac{2F}{V}(T_{in} - T_{out}) + \frac{2Q}{V\rho c_p} - \frac{dT_{in}}{dt} \quad (7)$$

### **Constrained inverse and IMC**

In this model the manipulated variable ( $Q$ ) directly affects the first derivative of the controlled variable, thus the system is of relative first order. To obtain the constrained inverse (Szeifert et al., 2007) a control specification needs to be set up, preferably of the same order as the relative order of the system:

$$\tau_c \frac{dT_{out}}{dt} + T_{out} = T_{ref} \quad (8)$$

If we substitute the derivative from the model equation (7), and rearrange, the result is an algebraic equation for the manipulated variable:

$$Q = \frac{V\rho c_p}{2} \left( \frac{T_{ref} - T_{out}}{\tau_c} + \frac{dT_{in}}{dt} \right) + F\rho c_p(T_{out} - T_{in}) \quad (9)$$

We can see it in this control equation that the last term describes steady state manipulation, and first two terms add the dynamic compensation. The measured disturbances are included, which is clearly a feed-forward element of this strategy. In the formula the value of the outlet temperature is calculated using the model of the system.

A clearly feed-forward strategy would not ensure the elimination of steady state error. Hence an IMC structure is used for model error compensation. Such a structure is also suitable for dead-time compensation. It must be noted that the measurement devices and manipulators have their own dynamic characteristics, which we have to take into consideration when constructing an IMC structure. The scheme for the controller is illustrated on *Figure 2*.



**Table 1****Identified black box models of measurements and manipulators**

Thermometer ( $T_{out}$ ) (1)	Heater signal ( $Q$ ) (2)	Flow rate signal ( $F$ ) (3)
$G(s) = \frac{1}{9.51s + 1}$	$G(s) = e^{-11.89s}$	$G(s) = e^{-3.50s}$

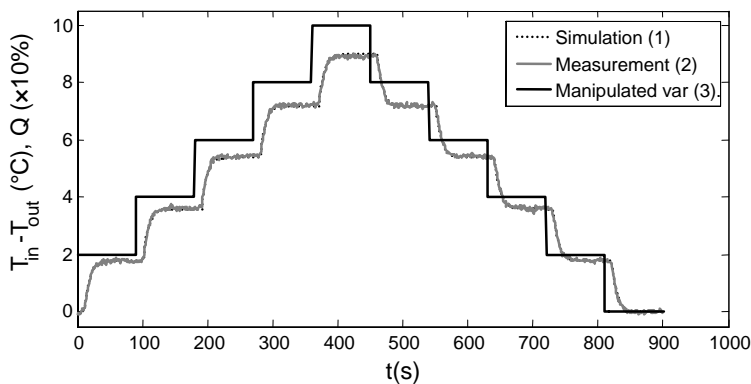
1. táblázat: A mérő- és beavatkozó eszközök illesztett fekete-doboz modelljei

Hőmérő  $T_{ki}(1)$ , Fűtőszál beavatkozó jel(2), Térfogatáram jel(3)

It is notable that the dead time of the manipulator is greater than that of the disturbance. This causes that after a step change in the disturbance the manipulator does not have any effect on the output, with no respect to the controller strategy used.

**Black box modeling**

The usual approach in control applications is black box modeling. To create a black box model measurement data is needed to identify its parameters (Figure 4). A series of measurements were carried out, in each the heater power, the manipulated variable, changed step by step, while the flow rate was constant. After each measurement a first order time delay model was fitted where the temperature rise (difference of outlet and inlet temperature) was the output and heater power signal was the input. These steps were repeated at other flow rates, and the parameters of the model (gain, time constant, dead time) were collected and plotted on a graph as the function of the flow rate (Figure 5, Table 2). Actually the signal obtained from the orifice plate has been used directly, instead of the calculated flow rate.

**Figure 4****Open loop experiment for identification of black box model**

4. ábra: Nyitott körű vizsgálat fekete-doboz modell identifikációjához

Szimuláció(1), Mérés(2), Beavatkozó jel(3)

Figure 5

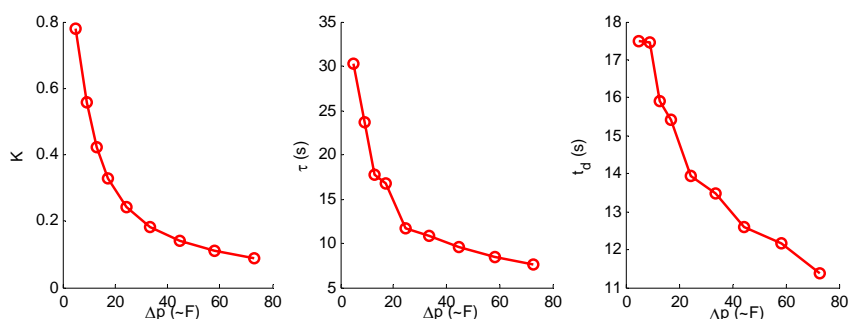
Black box model parameters as functions of  $\Delta p$ 5. ábra: Fekete-doboz modell paramétereinek függése  $\Delta p$ -től

Table 2

Relations of black box model parameters to  $\Delta p$  and calculation of  $F$ 

$K = 3.36\Delta p^{-0.83}$	$\tau = 71.87\Delta p^{-0.53}$	$t_d = 24.43\Delta p^{-0.17}$	$F = 5.184\Delta p + 15.47$
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2. táblázat: A fekete-doboz modell paramétereinek összefüggése  $\Delta p$ -vel és  $F$  számítása

It is clear that there is significant change in the behavior of the system due to flow rate. The gain is almost proportional to the -1 power of  $\Delta p$ , which can be justified on basis of first principle knowledge. The ratio of the minimal and maximal measured values is large enough to justify an adaptation strategy in feedback control.

## PI control and programmed adaptation

For feedback control a constrained PI controller (Abonyi *et al.*, 2005) has been used. For the first try its parameters were fixed and calculated with ITAE method using the average of obtained model parameters. The control performance was so poor that no further investigation was needed to say: pure feedback control is unable to handle disturbances in the water heating system.

For the second attempt the parameters of the PI controller were set on-line by ITAE and direct synthesis methods. The measured flow rate has been used for calculating the model parameters on-line, and then PI parameters were calculated.

It may be a question if this concept is feedback or feed-forward. It is a mixture of both: obviously the PI controller is a feedback one, but as the parameters are depending on a measured disturbance, the manipulated variable is able to immediately react to the changes in the disturbance, which is a feed-forward quality.

## Case studies

In the following section a set of reference and disturbance signals are applied in simulation and measurements.

### Simulation studies

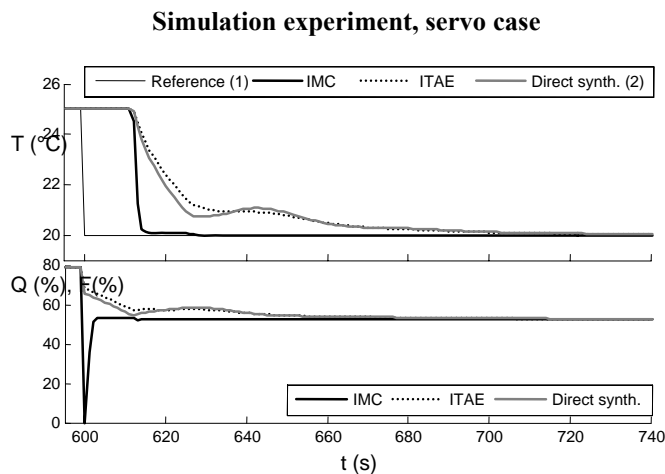
First simulation has been used to test the controller algorithms. The first principle model of the process has been used as controlled object, and a series of reference temperature and flow rate steps were used to evaluate controller performance.

In simulation studies the first principle model based controller has been superior to PI controllers of both tuning methods. This result is not surprising, as exactly the same model has been used as controlled object and reference model. Truly this is a feed-forward control, as no model error is fed back in the IMC structure.

In the servo case experiment the set point changes, while the disturbances remain on their previous values (*Figure 6*). The inverse controller acts faster, because dead time is compensated. The adaptive PI controllers act slower, they do not use the whole available manipulator capacity. Anyway all of them act immediately when the set point is changed. Moreover the PI controller tuned by direct synthesis shows some oscillations before reaching the set point, which is unwelcome characteristic in control.

In regulatory case the flow rate was changing while the set-point was constant (*Figure 7*). After the dead time of the disturbance had passed, the first changes in the controlled variable were the same for all control strategies, as the manipulator has greater dead time than the disturbance. All the controllers acted immediately, as the feed-forward element takes into consideration the measured disturbance. On the other hand the PI controllers had not continued to change the manipulated variable until observable error appeared on the controlled variable. They use the measured value of the output signal that does not belong to the same time as the manipulated variable, instead it is shifted with a dead time. The great difference at the inverse based controller, is that the dead time is compensated, and the effect of the manipulation is instantly observable on the model output, thus there is no time shift between the signals used to calculate the manipulated variable. The consistency of timing reduces the possibility of instability.

**Figure 6**

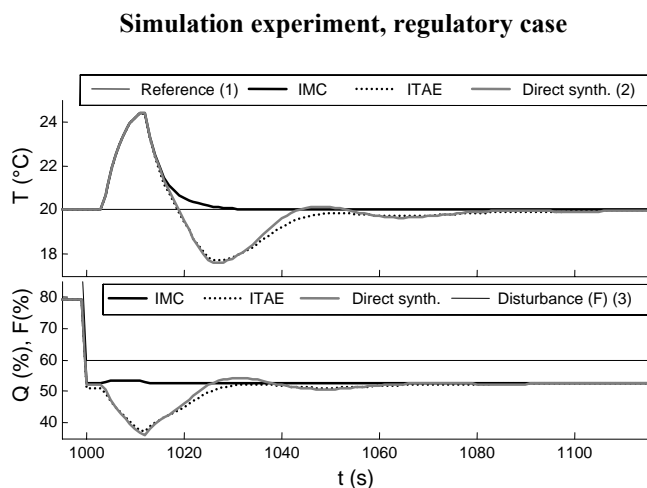


6. ábra: Szimulációs vizsgálat, szervó eset

Alapjel(1), Közvetlen szintézis(2)



Figure 7



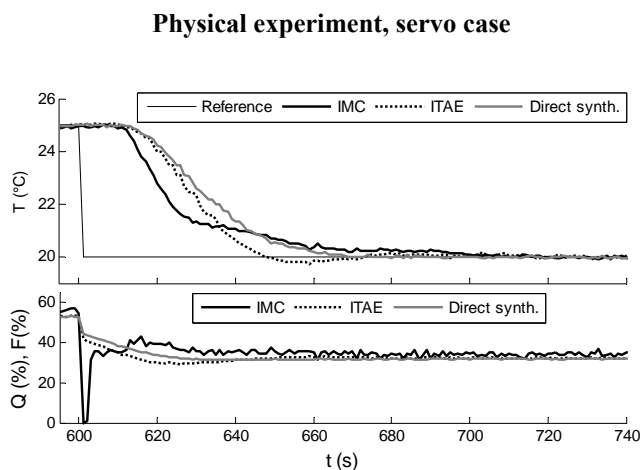
7. ábra: Szimulációs vizsgálat, zajkompenzációs eset

Alapjel(1), Közvetlen szintézis(2), Zavaró jel (3)

### Physical measurements

The same reference and disturbance signals were used also in a physical measurement (Figure 8, Figure 9). The simulation predicted most of the observed effects well, but here model error was also present.

Figure 8

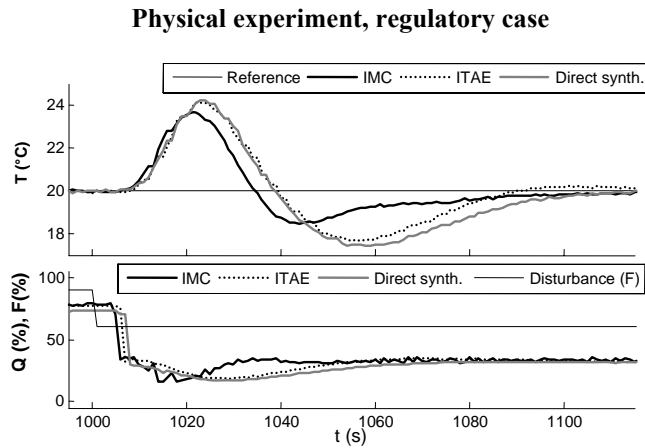


8. ábra: Fizikai vizsgálat, szervó eset

Lásd 6. ábra

The superiority of the inverse-based controller is not clear from the measurements. The set point change case differs from the simulation results. Though the inverse-based controller is still faster at the beginning, it continues to settle slowly. This may be the result of model error and its mild feedback compensation. The PI controllers also seem to be slower than in simulation, with slightly different characteristics. In their case the parameters of the black box model are calculated from the noisy differential pressure signal. This may also result model error, and different PI parameters than the simulation uses. Among the PI controllers the ITAE method is faster, but has a slight overshoot. As it is comparable to the measurement noise, it is negligible.

**Figure 9**



9. ábra: Fizikai vizsgálat, zajkompenzációs eset

Lásd 7. ábra

The regulatory case for PI controllers resembles to the simulation, with generally slower settling, and smaller oscillations. The difference in the inverse-based controller is significant, because there are two oscillations: the first is due to the dead time differences mentioned above, the second is due to model error and its compensation. The controlled variable settles in about the same time, as in the case of PI controllers, but with smaller peaks of swinging. The main advantage seems to be the dead time compensation of this strategy.

## CONCLUSIONS

We have seen the modeling and controller synthesis for water heating laboratory device. The two approaches used both had feedback and feed forward elements, as pure feedback had too poor control performance. The programmed adaptation PI controllers focus on feedback, and the feed forward is hidden in the PI parameter calculations. The IMC structure containing a constrained inverse focuses on feed forward, and there is only mild feedback in the model error compensation.

Using measured disturbance signal in the calculation of the manipulated variable improves controller performance. Utilizing correct a priori information in the synthesis of the controller also improves the final performance. The first principle and the black

box models are both vulnerable to latency and noise of measurements. A first principle model also needs to have some degrees of freedom by it can be fitted on measurement data. The structure of the model needs to be chosen carefully, as a well fitted model can be misleading, if its structure is not appropriate. In our case a multiple input system was simplified to a SISO system by the black box model.

The main advantage of the IMC structure was the dead time compensation. This is clear when simulation prevents model error, but in the measurements the model errors, especially dead time mismatch, significantly decreases the performance of the inverse-based controller. The feed forward by the inverse function makes the system to behave as it is prescribed, but model error flaws this strategy, as the feedback compensation is not in focus.

To summarize this work and draw conclusions we can state that any of the control strategies that used feed forward outperformed the pure feedback of the fixed parameter PI controller. Besides the three studied controllers the inverse-based IMC has proven to be the most pleasing. It should be noted that measurement studies show the real performance, and simulation studies usually show better performance.

### ACKNOWLEDGEMENT

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