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Investigations of Gears Geometry Using the Surface Constructor Kinematical Modelling System

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ABSTRACT

In the area of production of machines there is a need for high quality gears that work with improved efficiency. The goal of the machine designers is to improve the characteristics of existing types of gearings or invent new ones. The paper introduces a powerful new theory, the "Reaching Model", for generating enveloped kinematical surfaces that connect in spatial curves or to develop surface pairs connecting in a point-like manner because of the applied theoretical intermediary generating surface. The model and the software application based on it proved its suitability for gear surface development many times. The application implementing the theory provides advanced 3D visualization capabilities for displaying and animating kinematical surfaces, co-ordinate systems and other space attributes. This application has the Surface Constructor (SC) name and sets the goal of providing maximum freedom in modelling different kinematical surfaces and arrangements. For the entering of surfaces and motions, expressions can be used and symbolic algebraic computations carried out automatically behind the scenes. This complex application models the kinematical modelling process itself and can handle practically limitless types of gearings.
(Keywords: gear, kinematic modelling, Reaching Model)

ÖSSZEFOGLALÁS

Fogaskerék geometriai kutatások a Surface Constructor kinematikai modellező rendszer felhasználásával
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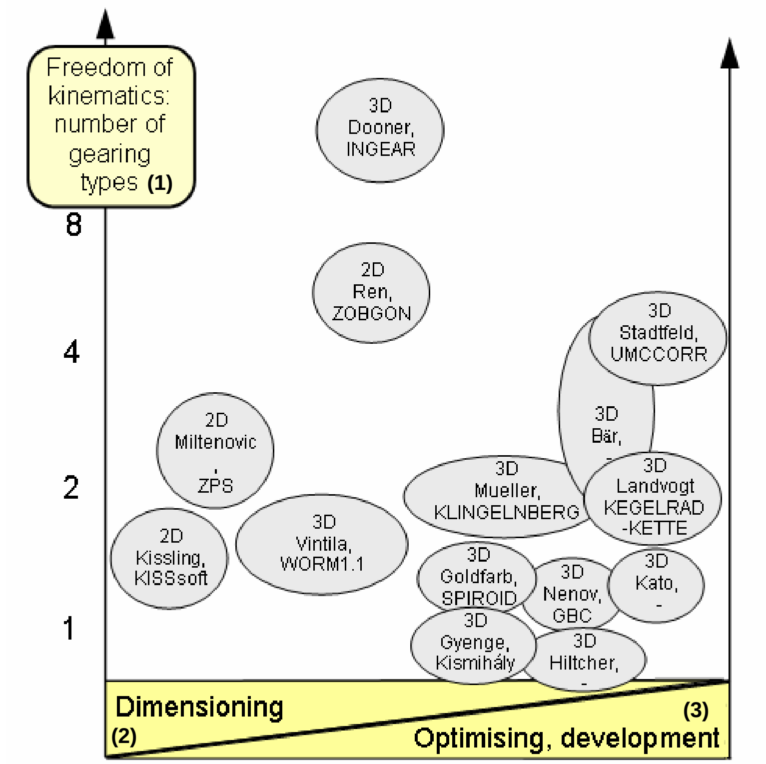
A gépgyártás területén igény jelentkezik magas minőségű fogaskerekekre, melyek növelt hatékonysággal üzemelnek. A géptervezők célja a meglévő hajtástípusok jellemzőinek javítása, vagy új típusok feltalálása. A dolgozat egy erőteljes új elméletet mutat be, az „Elérés Modell"-t, térbeli görbében kapcsolódó burkolt kinematikai felületek generálására, vagy az alkalmazott elméleti közvetítő származtató felület miatt pontszerű módon kapcsolódó felületpárok kifejlesztésére. A modell és a ráépülő szoftver alkalmazás sok esetben bizonyította az alkalmasságát fogaskerékfelületek fejlesztésére. Az elméletet megvalósító alkalmazás fejlett 3D-s megjelenítési képességeket nyújt a kinematikai felületek, koordinátarendszerek és egyéb térjellemzők láthatóvá tételére. Ez az alkalmazás a Surface Constructor (SC) névvel bír és a különféle kinematikai felületek és elrendezések modellezésében a maximális szabadság nyújtását tűzte ki célul. A felületek és mozgások megadására kifejezések használhatók és a háttérben automatikus szimbolikus algebrai számítások zajlanak. Ez a komplex alkalmazás magát a kinematikai modellalkotó folyamatot modellezi és gyakorlatilag korlátlan hajtástípust képes kezelni.
(Kulcsszavak: fogaskerék, kinematikai modellezés, Elérés Modell)

INTRODUCTION

The gears are one of the most complicated parts of the machines. Their efficiency and optimum design determines the success of the machine. Because of this, the computer aided development and optimization was always in the interest of the designers. The first level of innovation is the geometry. The geometry determines the possible usage area of the gearing and influences the manufacturability and working parameters of the gear. Consequently, the improvement and innovation of gear geometry was always in the focus of the interest of gear manufacturers. Because of these reasons many gear-CAD software have been developed. *Figure 1* gives a comparison of several programs depending on the suitability for dimensioning or development. The design tool of *Miltenovic* (1999) or *Kissling* (1999) is mainly intended for determination of the suitable sizes for a given gearing type to fulfil the construction requirements. The program of *Stadtfeld* (1999), *Landvogt* (1999) or *Kato* (1999) is more suitable for development of new characteristics of gearings or for optimisation.

Figure 1

Comparison of gearing design programs



1. ábra: Fogaskerék tervező programok összehasonlítása

Szabadságfok: hajtástípusok száma(1), Méretezés(2), Optimálás, fejlesztés(3)

The vertical axis measures the number of different modelled gear types. One of the programs having the largest freedom of embedded gearing kinematics is the INGEAR *Dooner* (1999). Most of the programs are intended to handle a few concrete gear types and the number of modelled sorts is limited. This recognition motivated the author to find a robust connection theory and to create a surface and kinematical relation independent gearing development tool. The name of this connection theory is the "Reaching Model". The original theory had been introduced with full particulars by the author *Dudás* (1996) and here only a brief description will be given of the essence and some capabilities of the model.

THE REACHING MODEL

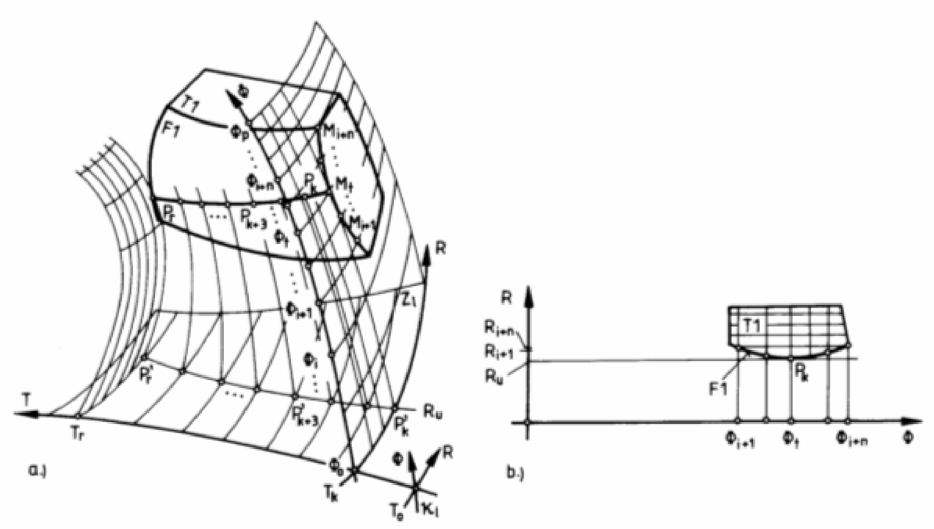
The author set the goal to develop a gearing investigation tool with less complexity of analysing capability but more freedom in constructing different kinematical arrangements. This freedom was realised by the robustness of Reaching Model theory and by use of inbuilt symbolic algebraic computation.

The main advantage of the model is its simplicity. In this model the generation of one of the points of the *F2* surface is equal to solving a simple minimum value problem. The denominative reaching process will be introduced briefly with the help of *Figure 2*. The Reaching Model applies a special non-Descartes co-ordinate system κ. The Φ co-ordinate lines as well as the co-ordinate axis Φ itself coincide the motion paths of the points of a surface that is in the co-ordinate system *K2* so Φ has two roles:

1. motion (time) parameter,
2. one of the three space co-ordinates of co-ordinate system κ.

Figure 2

The explanation of the reaching process



2. ábra: Az elérési eljárás magyarázata

Part a., of the figure shows the *T1* body in 3 dimensions. Part b., shows the *R*-Φ co-ordinate surface after transformed into a Descartes co-ordinate system. In the reaching process we choose a Φ co-ordinate line that does not intersect the *T1* body. Stepping from one Φ co-ordinate line to another going in *R* direction, the generating Φ co-ordinate line will be that which can reach *F1* first. This Φ line is the path of moving of point P_k' that will be one surface point of the generated surface *F2*. Point P_k will be the contact point.

The necessary condition of connection in the Reaching Model is

$$\frac{\partial R}{\partial \Phi} = 0\,. \tag{1}$$

where $R = R(\Phi,\ T,\ Z)$ is the reaching-coordinate function, Φ is the motion-path co-ordinate, T is the division co-ordinate in the κ slicing co-ordinate system, Z is the identifying parameter of the κ co-ordinate system. This necessary condition is equivalent to the $\boldsymbol{n}\cdot\boldsymbol{v}^{1,2} = 0$ condition of the known kinematical method.

The sufficient condition of the local minimum in the $\Phi = \Phi_t$ point that is equivalent to the real connection at the same location is given in the following form:

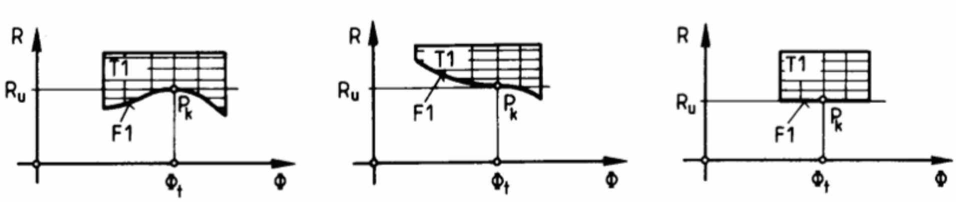
$$\left.\frac{\partial^v R}{\partial \Phi^v}\right|_{\Phi=\Phi_t} = 0 \qquad (v = 1, 2,\ldots, w\text{-}1) \text{ and} \tag{2}$$

$$\left.\frac{\partial^w R}{\partial \Phi^w}\right|_{\Phi=\Phi_t} > 0 \qquad \text{where } w \text{ is an even number.} \tag{3}$$

The model can give all types of local undercuts – maximum point, inflexion point, continuous connection, shown in *Figure 3* – simply by discussing the minimum value problem in a local tangential point.

Figure 3

Different types of local undercuts



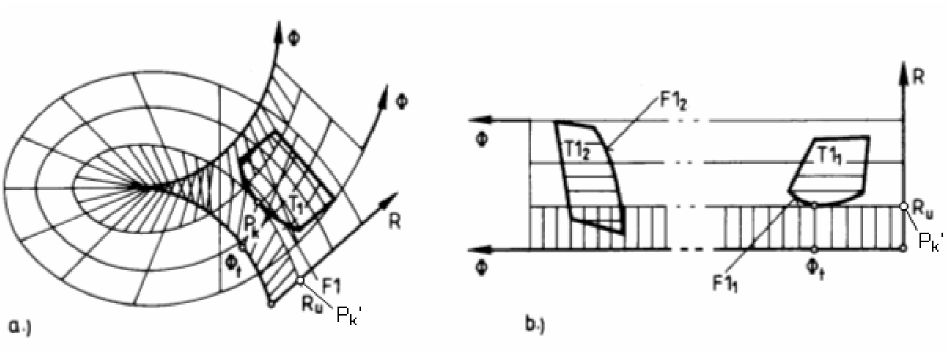
3. ábra: A lokális alámetszések különféle esetei

Moreover it can produce global cuts using the time and appropriate space interval given by the kinematical task, as *Figure 4* shows.

Figure 4a shows the real curved *R*-Φ co-ordinate system while *Figure 4b* shows it after transforming it to a Descartes co-ordinate system.

Figure 4

The generated P_k' point is destroyed later by the global cut



4.ábra: A generált P_k*' pontot később a globális elmetszés megsemmisíti*

Though the theory works with analytical expressions and partial derivatives, a robust, surface-independent software for realisation of the theory was developed on a discrete numerical basis.

THE SURFACE CONSTRUCTOR GEARING DEVELOPMENT TOOL

The developed application uses three levels for representation of the objects – curves, surfaces and determined results –, the top level uses symbolic algebraic representation of the entered generating curve or surface and the input kinematical relations. The middle level, after data entry represents these objects in numerical form. The lowest level presents these and the determined objects by visualisation.

The program can handle more derivation at the same time, but the generating surface has to be the same for all derivations.

The theory proved its suitability for gear surface development in many times, for example determining the gear surface and contacting patterns in case of a worm gearing having elliptical profile in the axle plane of the worm, determining the exact grinding wheel surface for grinding a Spiroid or globoid worm, analysing the possibility of axial pitch modification for cylindrical worm gearings, modelling special cam mechanisms, and so on. The application realizing the theory applies advanced 3D visualization capabilities for displaying surfaces, co-ordinate systems and other space attributes like velocity and acceleration vectors, curvature relations and the instantaneous axes of the motion, see *Dudás* (2007). The working of gear elements can be simulated and analyzed.

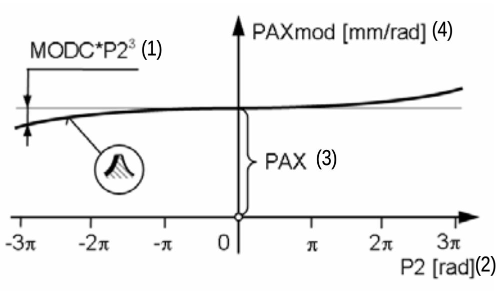
This application, named Surface Constructor (SC), sets the goal of providing maximum freedom in modelling contacting kinematical surfaces that work in different kinematical arrangements. For the entering of curves and surfaces expressions can be used and symbolic algebraic computations for vector-matrix and matrix-matrix multiplications carried out automatically behind the scenes. The complex application models the kinematical modelling process itself and can model practically limitless types of gearings. In the followings more emphasis will be laid on introduction of different resolved tasks accomplished by SC.

Example tasks resolved by SC

The first example introduces a new type of worm gearing having double-localised bearing pattern. The first localisation applied pitch modification along the length of the worm modifying the pitch by a cubic function, see *Figure 5*. This method localised the contact to the middle part of the worm providing smooth entering of teeth into the connection.

Figure 5

The cubic function of the modified axial pitch of the helicoid



5. ábra: A csavarfelület módosított axiális osztásának köbös függvénye

Konstans a modifikálás finomhangolására(1), Forgásparaméter(2), Tengelyirányú emelkedés(3), Tengelyirányú emelkedés köbös függvénye(4)

In the figure P2 is the rotational parameter of the helicoid surface generation, PAX is the axial pitch of a non-modified worm, PAXmod= MODC*P2^3 is the cubic function of the axial pitch. MODC is a constant for fine tuning the size of the modification. The thinner teeth at the two ends of the worm can be compared to the reference helicoid having uniform pitch along the length of the worm, see *Figure 6*.

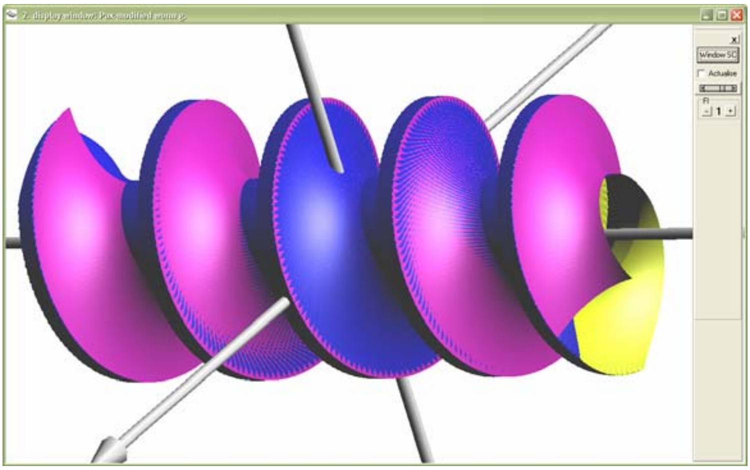
The second modification method, used on the same worm, localised the contact area to the middle region of the height of the tooth in radial direction. To realize this modification, the original ellipse profile of the worm was substituted with a larger one. By changing RO2 and GAMMA, the localisation can be fine tuned, as can be seen in *Figure 7*.

Comparing the double-modified worm to the reference helicoids the effect of the second modification can be observed, see *Figure 8*.

It is possible to generate the conjugate worm gear tooth surface using the non-modified reference helicoid and to analyse the connection between the worm gear and the double-modified worm. The different moments of the connection and the contact patterns can be seen in *Figure 9*. Figures show that the modification was successful, the connection of this type of gearing can tolerate little manufacturing and/or assembly errors, the contact pattern will remain in the internal surface region of the gear teeth. *Figure 10* shows the SC system in the midst of the development of the double-modified worm gearing.

Figure 6

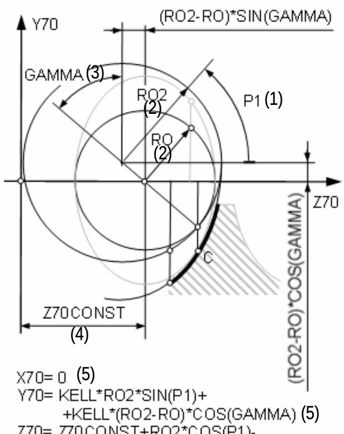
Comparing the modified worm and the reference helicoids



6. ábra: A módosított csiga és a referencia csavarfelület összevetése

Figure 7

Using a larger ellipse arc to localise the connection in radial direction of the worm



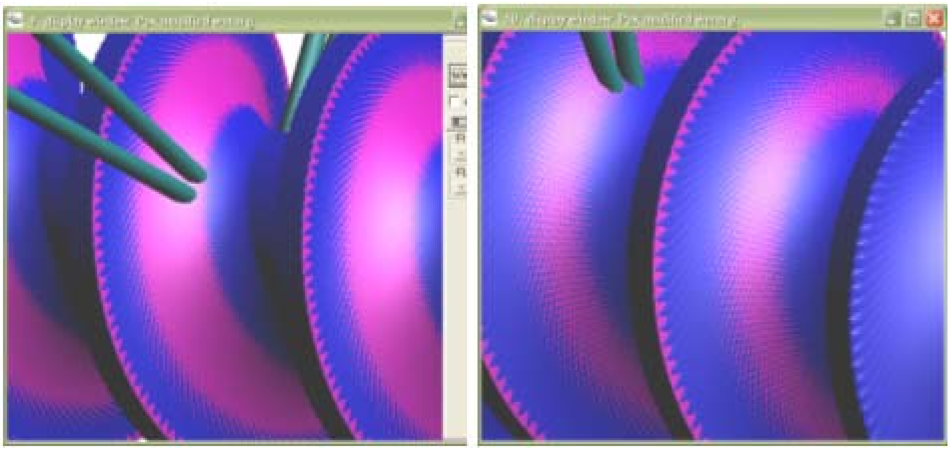
KELL: Elliptical constant (*Ellipszis konstans*)

7. ábra: Egy nagyobb ellipszis ív használata a kapcsolódás lokalizálására a csiga sugár irányában

Csiga felületi parameter(1), Profilellipszis alapkörsugár(2), Lokalizálás helyzetszöge(3), Profilelhelyező konstans(4), Koordináta irányok(5)

Figure 8

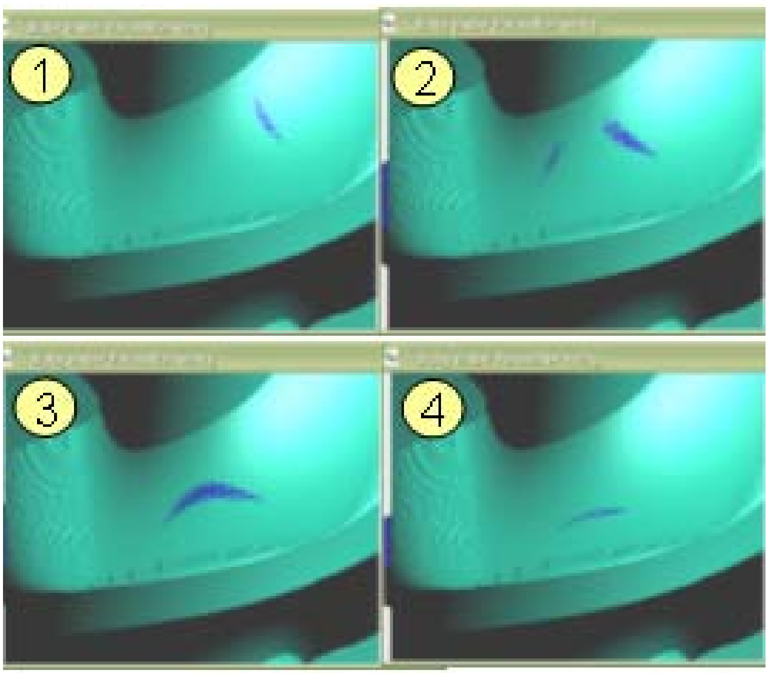
The effect of RO2-RO difference change



8. ábra: A RO2-RO különbség változtatásának hatása

Figure 9

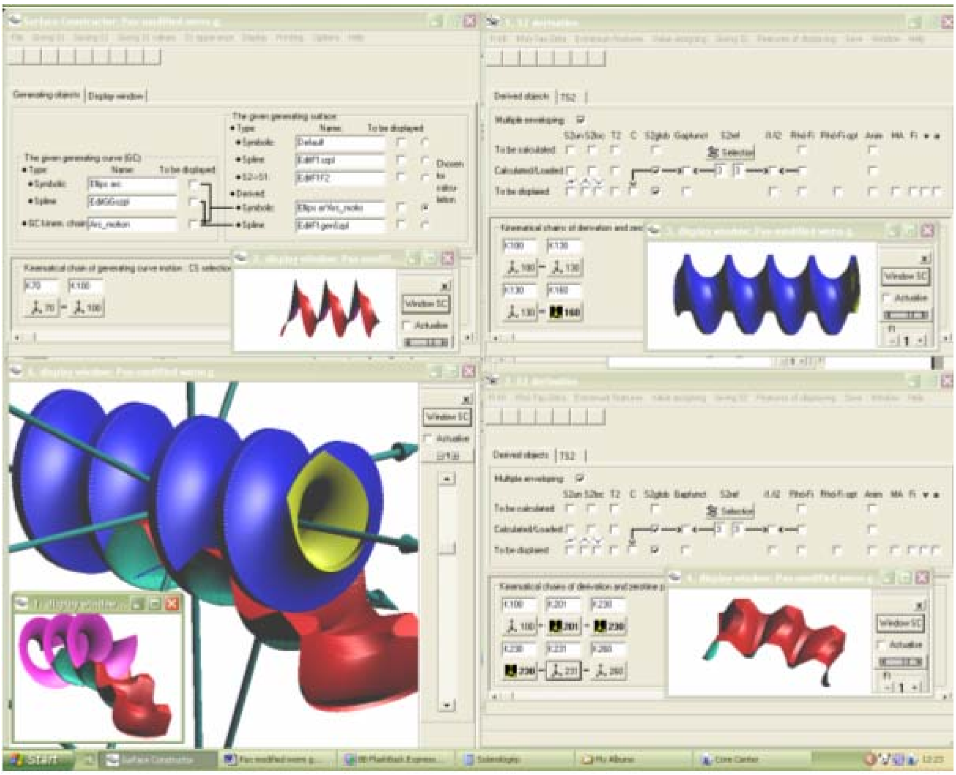
The contact patterns of the localised worm gearing



9. ábra: A lokalizált csigahajtás érintkezési mintázatai

Figure 10

The SC system in the midst of the development



10. ábra: Az SC rendszer fejlesztés közben

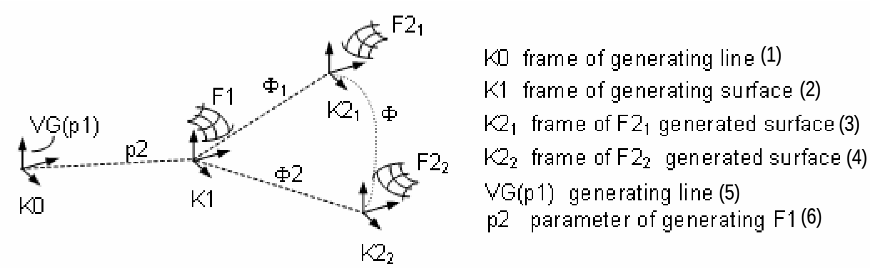
The second task resolved by SC program generated the surfaces of the members of a hypoid gearing using an intermediary generating cone surface. There is a contact line between the cone and the hypoid gear. After connecting the gears an ellipse-like contact pattern rises from the intersection of the mentioned theoretical contact lines, as shown in *Figure 11* and *Figure 12B*.

The animation of the motions makes possible the analysis of the enveloping process, the form and the wandering of the contact pattern. Because of the intermediary generating surface this hypoid gearing has an advantageous ellipse-like contact area (*Figure 12*).

The third example introduces a special capability of the program: beyond the determination of enveloped surface the application is capable to determine surfaces resulted by series of global cuts of discrete edges, e.g. edges of a hob. It is known that the gear surface cut by the edges of a hob has geometrical error because of a limited number of cuts and the geometrical errors decrease the efficiency of power transmission, see *Bercsey* (1999). The next example presents the determination of the dependency of this error on the number of the edges for a given hob. In the example the hob is conical and generates the crown gear of a Spiroid gearing. The kinematical arrangement of the manufacturing is given in *Figure 13*. K100 holds the gear and the hob is fixed to the K70 co-ordinate system.

Figure 11

Generating the hypoid surfaces $F2_1$ and $F2_2$ by F1 theoretical cone surface



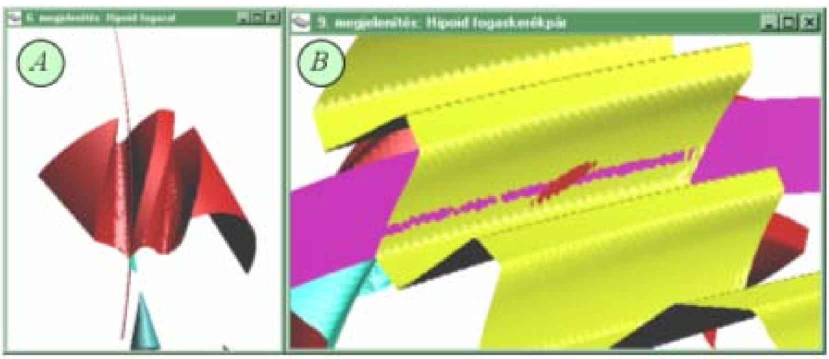
Φ – Connection parameters (*Kapcsolódás paramétere*), Φ1, Φ2 – Generating movement parameters (*Generáló mozgások paraméterei*)

11. ábra: Az F21 és az F22 hipoid felületek generálása az F1 elméleti kúpfelülettel

*Generáló görbe koordinátarendszere(1), Generáló felület koordinátarendszere(2), Generált felület koordinátarendszere(3), Generált felület koordinátarendszere(4), Generáló görbe (*p1 *– generáló görbe paramétere)(5), Generáló F1 felület paramétere(6),*

Figure 12

***A*: Generating the pinion, the cone seems to be a circle arc. *B*: The contact line of F1 and $F2_2$ and the ellipse-like contacting pattern between $F2_1$ and $F2_2$ hypoid gears**



12. ábra: A: A nyeles kerék generálása, a kúp egy körívben látszik. B: Az F1 és az F22 érintkezési vonala és az ellipszis-szerű érintkezési mintázat az F21 és az F22 hipoid fogaskerekek között

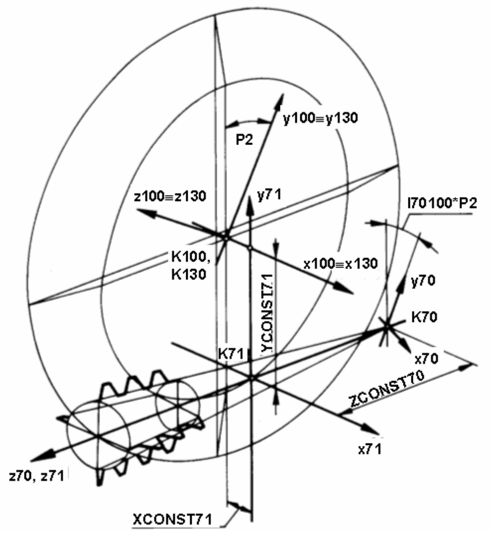
To determine how the error depends on the number of teeth of the hob the manufacturing with the next hobs was modelled:

Number of teeth: 15, 22, 29, 36, 43
Angle between teeth: 90, 60, 45, 36, 12 (degree)

The manufactured tooth surface in case of a hob having 15 edges and the swept surface of one of the edges can be seen in *Figure 14*. Small arrows show the locations of maximal errors caused by remaining material.

Figure 13

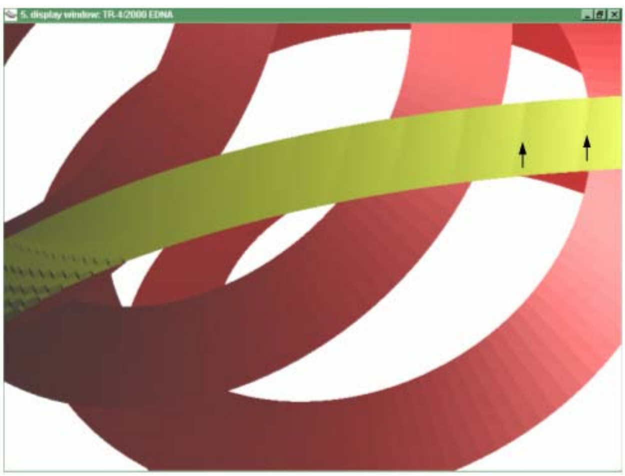
The kinematical relations of the co-ordinate systems at hobbing



13. ábra: A koordináta rendszerek kinematikai viszonya a lefejtőmarásnál

Figure 14

The manufactured Spiroid gear tooth surface and the track of an edge

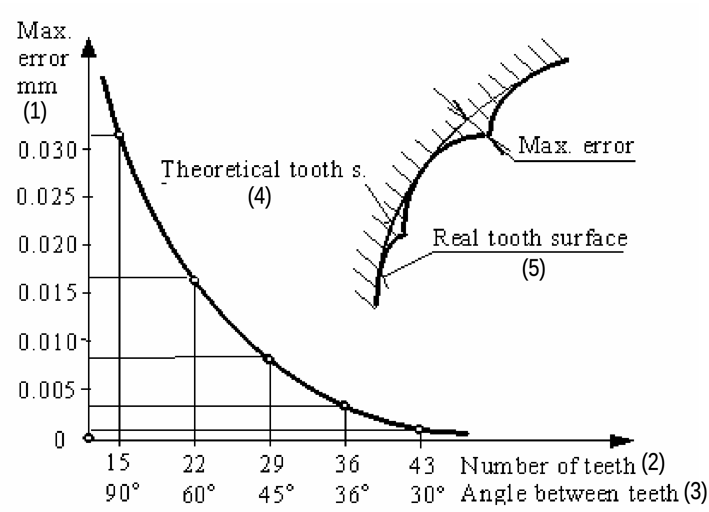


14. ábra: A megmunkált Spiroid fogaskerék fogfelülete és egy él nyoma

After accomplishing all the experimental modelling of the hobbings, the error function could be drawn, see *Figure 15*. The introduced technique can be used to determine the required minimum number of teeth of the hob for another concrete Spiroid gearing.

Figure 15

The maximum error function



15. ábra: A maximális hiba függvény

Maximális hiba, mm(1), Fogak száma(2), Fogak közötti szög(3), Elméleti fogfelület(4), Valós fogfelület(5)

CONCLUSION

The paper presented a new theory and application for modelling all types of gearings. The main advantage of the theory is its simplicity. It can detect all types of locale undercuts and the issue of global cut. Among the novelties of the system can be mentioned the integrated symbolic algebraic computation, aiding gear development without inbuilt kinematical arrangements, practically limitless kinematical modelling capability and modelling of the kinematical modelling task itself.

In the first example an axially modified worm gearing was introduced. The second example dealt with generation of the two members of a hypoid gearing using an intermediary generating cone surface. The modelled hypoid gear pair had an ellipse-shaped contact pattern. The third example demonstrated the capability of modelling the hobbing process where the geometrical error of the manufactured tooth surface depends on the number of cutting edges.

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