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Kaposvári Egyetem, Állattudományi Kar, Kaposvár
University of Kaposvár, Faculty of Animal Science, Kaposvár

Identification and Analysis of MIMO Systems based on Clustering Algorithm

B. Feil, J. Abonyi, J. Madár, S. Németh, P. Árva
University of Veszprém, Department of Process Engineering, Veszprém, H-8201 P.O. Box 158

ABSTRACT

This paper presents a compact Takagi-Sugeno fuzzy model that can be effectively used to represent MIMO dynamical systems. For the identification of this model a modified Gath-Geva fuzzy clustering algorithm has been developed. The case studies demonstrate that the proposed algorithm can be a useful and effective tool to select the embedding dimension of a dynamical system. This is a key step toward the analysis and prediction of nonlinear and chaotic time-series. The clustering is applied in the reconstruction space defined by the lagged output variables. The main advantage of the proposed solution is that three tasks are simultaneously solved during clustering: selection of the embedding dimension, estimation of the intrinsic (local) dimension, and identification of a model that can be used for prediction. The results were excellent in the case of the analyzed, three and four dimensional systems. Programs and data sets will be available via Internet on our web page http://www.fmt.vein.hu/softcomp/timeseries.
(Keywords: MIMO model, clustering algorithm, state-space reconstruction, chaotic time series)

ÖSSZEFOGLALÁS

MIMO modellek identifikációja és analízise csoportosítási algoritmus segítségével
Feil B., Abonyi J., Madár J., Németh S., Árva P.
Veszprémi Egyetem, Folyamatmérnöki Tanszék, Veszprém, 8201 Pf. 158.

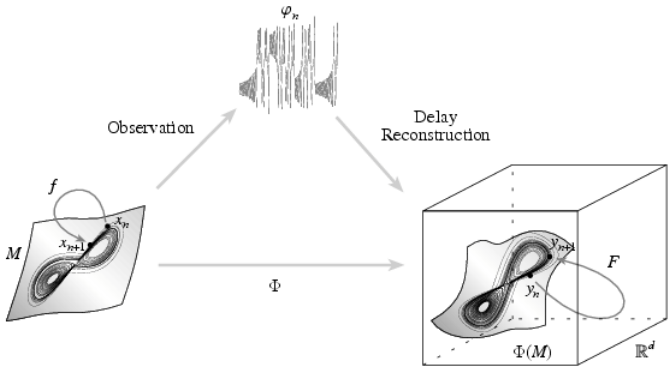
Nemlineáris több bemenetű-több kimenetű (MIMO) rendszerek modell-identifikációja fontos és kihívásokkal teli probléma. Fuzzy modellek hatékonyan alkalmazhatók komplex nemlineáris dinamikus rendszerek esetén, de többnyire egy bemenetű-egy kimenetű modellekre találunk példákat az irodalomban. Ez a cikk olyan kompakt Takagi-Sugeno fuzzy modell identifikációját mutatja be, amely dinamikus MIMO rendszereket is képes reprezentálni. Ennek a modellnek az identifikációjához fuzzy csoportosítási algoritmust fejlesztettünk ki. Az esettanulmány mutatja, hogy a javasolt algoritmus hasznos és hatékony eszköz dinamikus rendszerek dimenziójának meghatározásában is. Ez a lépés kulcsfontosságú nemlineáris és kaotikus idősorok analízise és predikciója esetén is. A csoportosítást a kimeneti változók időkésleltetett tagjai által definiált ún. rekonstrukciós térben alkalmaztuk. A javasolt módszer nagy előnye, hogy három feladatot lehet megoldani a csoportosítás alkalmazásával szimultán módon: a helyes és a lokális dimenzió meghatározását, továbbá a predikcióhoz is használható modell identifikációját. A lokális dimenzió a fuzzy kovariancia mátrixok sajátértékeinek analíziséből becsülhető, míg a helyes állapottér-dimenzió a csoportok által definiált lokális modellek predikciós teljesítménye alapján állapítható meg.
(Kulcsszavak: MIMO modell, csoportosítási algoritmus, állapottér-rekonstrukció, kaotikus idősorok)

INTRODUCTION

Most of the developments in the field of nonlinear dynamics over the past century have assumed that one had a complete description of the dynamical system under consideration. In principle the practical application of these results thus requires the simultaneous measurement of all the state variables. Unfortunately, in many real problems one has only the sketchiest information about what these variables are, and one certainly has no hope of observing them all. Instead, one typically has a time series of one or more observables of the system, whose relationship to the state variables is at best uncertain. Fortunately, a remarkable result discovered by *Takens* some twenty years ago shows in *Takens* (1981) that typically one can reconstruct the dynamics of an unknown deterministic finite-dimensional system from a scalar time series generated by that system.

Figure 1

Overview of delay embedding for deterministic systems



1. ábra: Determinisztikus rendszerek időkésleltetett beágyazásának áttekintése

Megfigyelés(1), Időkésleltetett rekonstrukció(2)

We denote the state space of our dynamical system as $\mathbf{M}$, which is assumed to be a finite-dimensional compact manifold. We assume that the state of the system at time k, denoted $\mathbf{x}_k \in \mathbf{M}$ evolves according to $\mathbf{x}_{k+1} = \mathbf{f}(\mathbf{x}_k)$. The system is observed using a smooth measurement function $y : \mathbf{M} \to R$ giving a scalar time series $y_k = \mathbf{g}(\mathbf{x}_k)$. The aim of the so-called method of delays is to reconstruct the state space $\mathbf{M}$ and the dynamics $\mathbf{f}$ from the time series y_k. Because $\mathbf{M}$ is high-dimensional and each component of y_k is only one-dimensional, it is clear that to obtain a suitable state space we need to somehow group different elements of the time series. The most natural (but not the only) way of doing this is to take successive y_k to create a vector:

$$\mathbf{y}_k = [y_k, y_{k-\tau}, ..., y_{k-\tau(d_e-1)}]^T \tag{1}$$

where τ correspond to the interval on the time series that creates the reconstructed state space (lag time), usually chosen to be the first zero of the autocorrelation function or the first minimum of mutual information. The mutual information between two

measurements is used to measure the generally nonlinear dependence of two variables. The mutual information for two measurements y_n and $y_{n+\tau}$ from the same time series is expressed by

$$I_n(\tau) = \log_2\left[\frac{P(y_n, y_{n+\tau})}{P(y_n), P(y_{n+\tau}),}\right] \tag{2}$$

where the individual probability densities, $P(y_n)$ and $P(y_{n+\tau})$, are equal to the frequency with which the data points y_n and $y_{n+\tau}$ appear in the time series, respectively. The frequency can be obtained directly through tracing the data points in the entire time series. The joint probability density, $P(y_n, y_{n+\tau})$, is obtained by counting the number of times the values of the y_n and $y_{n+\tau}$ pair are observed in the series. Average mutual information is computed for all data points in the following manner:

$$I(\tau) = \sum_{n=1}^{N-\tau} P(y_n, y_{n+\tau}) \log_2\left[\frac{P(y_n, y_{n+\tau})}{P(y_n), P(y_{n+\tau}),}\right] \tag{3}$$

When $P(y_n, y_{n+\tau}) = P(y_n)P(y_{n+\tau})$ such that $I(\tau)$ approaches zero, the data points y_n and $y_{n+\tau}$ are completely independent. In practice the first minimum of the average mutual information function is chosen as the lag time *Adeli* (2003).

The number of components d_e that we use is usually referred to as the *embedding dimension*. Although the data is embedded in a d_e dimensional space, this does not necessarily mean that it fills that space. Sometimes the system defines a nonlinear hyper-surface in which the state variables reside, i.e the tangent space. The dimension of this hyper-surface is often referred as intrinsic, topological or local dimension, d_l. *Takens* proved in *Takens* (1981), that ideal systems (an infinite number of points without noise) converge to the real dimension if $d_e \geq 2d_l + 1$. According to Sauer et. al. $d_e \geq d_l + 1$ in practical cases can be enough to reconstruct the original state space.

CLUSTERING IN THE RECONSTRUCTED SPACE

The bottleneck of the data-driven identification of *Takagi-Sugeno* (TS) models is the identification of the antecedent part (membership functions) of TS models that requires nonlinear optimization. Hence, for this purpose heuristic approaches, like fuzzy clustering methods, are often applied. The aim of this section is to propose a new clustering-based technique for the identification of the fuzzy model presented above.

Problem Formulation

The objective of clustering is to partition the identification data $\mathbf{Y} = [\mathbf{y}_1^T, ..., \mathbf{y}_N^T]$ into c clusters. The fuzzy partition is represented by the $U = [\mu_{i,k}]_{c \times N}$ matrix, where the $\mu_{i,k}$ element of the matrix represents the degree of membership, how the $\mathbf{y}_k$ observation is in the cluster $i = 1, ..., c$. Different cluster shapes can be obtained with different kinds of clustering algorithm based on different prototype, e.g., point or linear varieties (FCV) or with different distance measure. Mostly, the *Gustafson-Kessel* clustering algorithm is applied for the identification of TS models *Gustafson* (1979). A drawback of this algorithm is that only clusters with equal volumes can be found and the resultant clusters cannot be directly used to form membership functions. The *Gath* and *Geva* clustering (GG) algorithm does not suffer from these problems *Gath* (1989). *Gath* and *Geva* interpret the data as normally distributed random variables and based on this assumption they worked out an algorithm that is able to

determine the parameters of the clusters, hence the expected value (center) $\mathbf{v}_i$, covariance matrix $\mathbf{F}_i$ and *a priori* probability $p(\eta_i)$ of the *i*th cluster, $i = 1,...,c$ where c is the number of clusters. In *Abonyi* (2002) and *Kim* (1998) it has been shown how antecedent fuzzy sets defined on linearly transformed input variables can be derived form clusters obtained by the GG algorithm. This may, however, complicate the interpretation of the rules.

To from an easily interpretable model that does not use the transformed input variables, a new clustering algorithm has been proposed based on the Expectation Maximization (EM) identification of Gaussian mixture models *Abonyi* (2002). This approach can be extended to the supervised clustering based identification of fuzzy classifiers. The aim of this section is to show how this EM based identification technique can be extended to the identification of MIMO fuzzy models.

Clustering Algorithm

The clustering is based on the minimization of the sum of weighted squared distances between the samples, $(\mathbf{y}_{k+1}, \mathbf{y}_k)$ and the cluster prototypes, η_i.

$$J(\mathbf{Y},\mathbf{U},\eta) = \sum_{i=1}^{c}\sum_{k=1}^{N}(\mu_{i,k})^m D_{i,k}^2 \tag{4}$$

The basic idea of the proposed algorithm is to define the cluster prototype which contains three terms and inversely proportional to the probability of the data:

$$\frac{1}{D_{i,k}^2} = p(\eta_i)p(\mathbf{y}_k \mid \eta_i)p(\mathbf{y}_{k+1} \mid \mathbf{y}_k, \eta_i) = w_i \prod_{j=1}^{d_e} \exp\left(-\frac{1}{2}\frac{(y_{j,k}-v_{i,j})^2}{\sigma_{i,j}^2}\right) \cdot \frac{\exp\left(-\frac{1}{2}(\mathbf{y}_{k+1}-\mathbf{A}_i\mathbf{y}_k-\mathbf{b}_i)^T(\mathbf{P}_i)^{-1}(\mathbf{y}_{k+1}-\mathbf{A}_i\mathbf{y}_k-\mathbf{b}_i)\right)}{(2\pi)^{de/2}\sqrt{|\mathbf{P}_i|}} \tag{5}$$

The $p(\mathbf{y}_k \mid \eta_i)$ distribution is parameterized as Gaussian distributions *Gershenfeld* (1999), and defines the domain of influence of a cluster (so it is based on the $\mathbf{v}_i$ geometrical distance between the cluster center and the $\mathbf{y}_k$ vector). Since the simplicity and interpretability of the model is important, the cluster weighted covariance matrix is reduced to its diagonal elements, $\sigma_{i,j}^2$, which resembles the simplified axis-parallel version of the *Gath-Geva* clustering algorithm *Hoppner* (1999). The third distance term is based on the performance of the local linear models where $\mathbf{P}_i$ is the weighted covariance matrix of the modeling error of the *i* th local model.

The $\mu_{i,k} = p(\eta_i \mid \mathbf{y}_{k+1}, \mathbf{y}_k)$ weight denotes the membership of the sample that is generated by the i th cluster

$$p(\eta_i \mid \mathbf{y}_{k+1}, \mathbf{y}_k) = \frac{p(\mathbf{y}_{k+1}, \mathbf{y}_k \mid \eta_i)p(\eta_i)}{p(\mathbf{y}_{k+1}, \mathbf{y}_k)} \tag{6}$$

To get a fuzzy partitioning space, the membership values have to satisfy the following conditions:

$$U \in \mathbf{R}^{c\times N} \mid \mu_{i,k} \in [0,1], \forall i,k; \sum_{i=1}^{c}\mu_{i,k} = 1, \forall k; 0 < \sum_{k=1}^{N}\mu_{i,k} < N, \forall i \tag{7}$$

The minimization of the Equation (4) function represents a non-linear optimization problem that is subject to constraints defined by (7) and can be solved by using a variety of available methods. The most popular method, however, is the alternating optimization (AO), which is formulated as follows:

Initialization Given a set of data **Y** specify the number of clusters, *c*, choose a weighting exponent (usually *m*=2) and a termination tolerance $\varepsilon>0$. Initialize the partition matrix (randomly), $\mathbf{U}=[\mu_{i,k}]_{c\times N}$.

Repeat for $l=1,2,...$

Step 1 Calculate the parameters of the clusters

– Centers of the membership functions:

$$\mathbf{v}_i^{(l)}=\frac{\sum_{k=1}^{N}\mu_{i,k}^{(l-1)}\mathbf{y}_k}{\sum_{k=1}^{N}\mu_{i,k}^{(l-1)}},1\le i\le c \tag{8}$$

– Standard deviation of the Gaussian membership function:

$$\sigma_{i,j}^{2(l)}=\frac{\sum_{k=1}^{N}\mu_{i,k}^{(l-1)}(y_{j,k}-v_{i,j})^2}{\sum_{k=1}^{N}\mu_{i,k}^{(l-1)}},1\le i\le c \tag{9}$$

– Parameters of the local models.

The modeling framework that is based on combining a number of local models, where each local model has a predefined operating region in which the local model is valid is called *operating regime based model* [Murray, (1997)]. This model is formulated as:

$$\mathbf{y}_{k+1}=\sum_{i=1}^{c}\beta_i(\mathbf{y}_k)\underbrace{(\mathbf{A}_i\mathbf{y}_k+\mathbf{b}_i)}_{f_i}=\sum_{i=1}^{c}\beta_i(\mathbf{y}_k)[\mathbf{y}_k^T\,1]\boldsymbol{\theta}_i^T=\sum_{i=1}^{c}\beta_i(\mathbf{y}_k)\mathbf{y}_{k+1}^i \tag{10}$$

where the $\beta_i(\mathbf{y}_k)$ function describes the operating regime of the $i=1,...,c$-th local linear models, the local models are defined by the $\boldsymbol{\theta}_i=[\mathbf{A}_i\quad\mathbf{b}_i]$ parameter set and using this notation $\mathbf{y}_{k+1}^i=f_i(\mathbf{y}_k,\boldsymbol{\theta}_i)$ is the output of the ith local model. The output of this model is linear in the elements of the $\mathbf{A}_i$ consequent matrices and the $\mathbf{b}_i$ offset vectors. Therefore, these parameters can be estimated from the data by linear least-squares techniques. The N identification data pairs and the truth values of the fuzzy rules are arranged in the following regressor matrix $\mathbf{Y}$ and regressand matrix $\mathbf{Y}_{(+1)}$ where the subscript denotes that these matrices contain the same terms, $\mathbf{y}_k$, just shifted with one sample time.

$$\mathbf{Y}=[\mathbf{y}_1,\mathbf{y}_2,...,\mathbf{y}_N] \tag{11}$$

$$\mathbf{Y}_{(+1)}=[\mathbf{y}_2,\mathbf{y}_3,...,\mathbf{y}_{N+1}] \tag{12}$$

$$\boldsymbol{\beta}_i=\begin{bmatrix}\beta_i(\mathbf{y}_1) & 0 & \cdots & 0\\ 0 & \beta_i(\mathbf{y}_2) & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \cdots & \beta_i(\mathbf{y}_N)\end{bmatrix} \tag{13}$$

where the elements of the $\boldsymbol{\beta}_i$ matrix are equal to the memberships that the $\mathbf{y}_k$ sample is generated by the ith cluster $\beta_i(\mathbf{y}_k)=\mu_{i,k}^{(l-1)}=p(\eta_i\,|\,\mathbf{y}_{k+1},\mathbf{y}_k)$. By using this notation, the weighted least squares solution of $\boldsymbol{\theta}_i$ is:

$$\boldsymbol{\theta}_i=[\mathbf{Y}^T\boldsymbol{\beta}_i\mathbf{Y}]^{-1}\mathbf{Y}^T\boldsymbol{\beta}_i\mathbf{Y}_{(+1)} \tag{14}$$

– Covariance of the modeling errors of the local models.

$$\mathbf{P}_i = \frac{\sum_{k=1}^{N} (\mathbf{y}_{k+1} - f_i(\mathbf{y}_k, \boldsymbol{\theta}_i))(\mathbf{y}_{k+1} - f_i(\mathbf{y}_k, \boldsymbol{\theta}_i))^{\mathrm{T}} p(\eta_i \mid \mathbf{y}_{k+1}, \mathbf{y}_k)}{\sum_{\mathrm{k}=1}^{N} p(\eta_i \mid \mathbf{y}_{k+1}, \mathbf{y}_k)} \tag{15}$$

– *A priori* probability of the cluster

$$p(\eta_i) = \frac{1}{N} \sum_{k=1}^{N} \mu_{i,k}^{(l-1)} \tag{16}$$

– Weight (impact) of the rules:

$$w_i = p(\eta_i) \prod_{j=1}^{n} \frac{1}{\sqrt{2\pi\sigma_{i,j}^2}} \tag{17}$$

Step 2 Compute the distance measure $D_{i,k}^2$ by (5).
Step 3 Update the partition matrix

$$\mu_{i,k}^{(l)} = \frac{1}{\sum_{j=1}^{c} \left(D_{i,k} / D_{j,k} \right)^{2/(m-1)}}, 1 \le i \le c, 1 \le k \le N \cdot \tag{18}$$

until $\| \mathbf{U}^{(l)} - \mathbf{U}^{(l-1)} \| < \varepsilon$.

The remainder of this section is concerned with the theoretical convergence properties of the proposed algorithm. Since, this algorithm is a member of the family of algorithms discussed in *Hathaway* (1993), the following discussion is based on the results of *Hathaway* and *Bezdek*. Using Lagrange multiplier theory, it is easily shown that for $D_{i,k} \ge 0$, (18) defines $\mathbf{U}^{(l+1)}$ to be a global minimizer of the restricted cost function (4). From this it follows that the proposed iterative algorithm is a special case of grouped coordinate minimization, and the general convergence theory can be applied for reasonable choices of $D_{i,k}$ to show that any limit point of an iteration sequence will be a minimizer, or at worst a saddle point of the cost function J. The local convergence result in *Bezdek* (1987) states that if the distance measures $D_{i,k}$ are sufficiently smooth and a standard convexity holds at a minimizer $(\mathbf{U}^*, \eta^*)$ of J, then any iteration sequence started with $\mathbf{U}^{(0)}$ sufficiently close to $\mathbf{U}^*$ will converge to $(\mathbf{U}^*, \eta^*)$. Furthermore, the rate of convergence of the sequence will be w-linear. This means that there is a norm $\| * \|$ and constants $0 < \gamma < 1$ and $l_0 > 0$, such that for all $l \ge l_0$, the sequence of errors $\{e^l\} = \{\| (\mathbf{U}^l, \eta^l) - (\mathbf{U}^*, \eta^*) \|\}$ satisfies the inequality $e^{l+1} < \gamma e^l$.

ESTIMATION OF THE LOCAL DIMENSION

The collection of c clusters approximates the tangent space. Hence, the clusters can be approximately regarded as local linear subspaces described by the cluster ellipsoids as shown in *Figure 2*. The smallest eigenvalues λ_{i,d_e} of the cluster covariance matrices $\mathbf{F}_i$ that are typically in orders of magnitude smaller than the remaining eigenvalues *Babuska* (1998), *Abonyi* (2002).

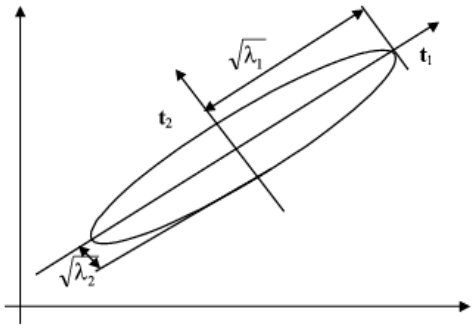
In *Cadzow* (1993) it has been shown that the widely applied minimum description length criterion (MDL) model order selection criterion can be expressed based on the smallest eigenvalues of the data covariance matrix. Applying the approximation that

$N^{1/N} \approx 1$ for large N, one can see that the MDL criterion asymptotically provides the same information as the minimum eigenvalue of the covariance matrix. The utilized fuzzy clustering obtains local linear approximation of the nonlinear system, MDL criterion can be modified for cluster-based model-order estimation by weighting the values of this simplified cost functions calculated from the cluster covariance matrices with the a priori probability of the clusters:

$$J^{d_e} = \sum_{i=1}^{c} p(\eta_i)\lambda_{i,\min} \tag{19}$$

Figure 2

Eigenvalues of clusters obtained by GG clustering



2. ábra: A csoportok sajátértékei GG csoportosításban

ESTIMATION OF THE EMBEDDING DIMENSION

In case of the proper number of embedding dimensions, the behavior of the time-series in the reconstruction space can be formulated by a smooth nonlinear function

$$\mathbf{y}_{k+1} = \mathbf{f}_r(\mathbf{y}_k) \tag{20}$$

While it may not be possible to find a model that is universally applicable to describe the unknown $\mathbf{f}_r(.)$ MIMO system, it would certainly be worthwhile to build local linear models for specific operating points of the process and one of these methods is proposed in this paper. The main advantage of this framework is its transparency, because the operating regimes of the local models can be represented by fuzzy sets *Babuska* (1997). This representation is appealing, since many systems change behavior smoothly as a function of the operating point, and the soft transition between the regimes introduced by the fuzzy set representation captures this feature in an elegant fashion. The proposed fuzzy model can be seen as a multivariable linear parameter varying system model (LPV). As this method forces the local linear models to fit the data locally, it does not give an optimal fuzzy model in terms of a minimal global prediction error, but it ensures that the fuzzy model is interpretable as a Linear Parameter Varying (LPV) system *Abonyi* (2000).

It has been shown that the clustering algorithm defines a function mapping $\mathbf{y}_{k+1} = \mathbf{f}_r(\mathbf{y}_k)$. If this mapping is similar to the original dynamical system $\mathbf{f}$, it is required that $\mathbf{y}_{k+1} = \mathbf{y}_{p+1}$ whenever $\mathbf{y}_k = \mathbf{y}_p$. In terms of the time series this condition amounts to

$\mathbf{y}_{n+de} = \mathbf{y}_{p+de}$ whenever $\mathbf{y}_k = \mathbf{y}_p, \ldots, \mathbf{y}_{k+de} = \mathbf{y}_{p+de}$ and is thus equivalent to the time series being perfectly predictable. In this context one might also ask about the regularity of $\mathbf{f}_r$, i.e. is it continuous, smooth etc? Hence, the embedding dimension is determined by increasing the number of lagged outputs, d_e, and performing the clustering and the analysis of the eigenvalues of the clusters and the one-step ahead prediction model of error.

APPLICATION EXAMPLES AND DISCUSSION

The proposed approach has been tested in several higher dimensional chaotic time-series and gave very convincing results. In every case study 15000 samples were used from variable x to reconstruct the state space, the number of the clusters $c = 20$, the termination tolerance $\varepsilon = 10^{-4}$ and the weighting exponent $m = 2$. All the programs were written in MATLAB.

The first system is Rössler attractor. The following three differential equations define this system:

$$\dot{x} = -(y + z) \tag{21}$$

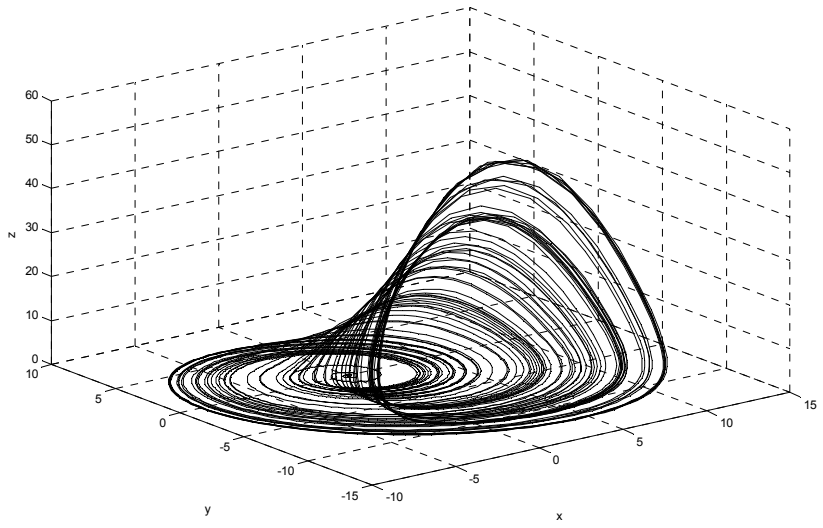
$$\dot{y} = x + ay \tag{22}$$

$$\dot{z} = b + z(x - c) \tag{23}$$

The initial conditions for solution of the differential equations are $x(0) = 0$, $y(0) = 0$ and $z(0) = 0$ and the used parameter values are $a = 0.38$, $b = 0.3$ and $c = 4.5$, following *Adeli* (2003). Under these conditions the trajectories can be given as depicted in *Figure 3*. The variable x can be shown in time in *Figure 4*.

Figure 3

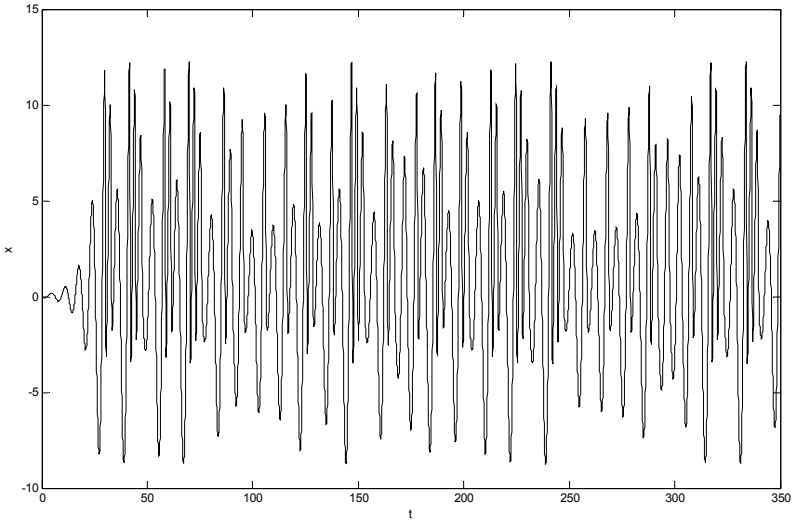
Trajectories in the original state space in case of the Rössler system



3. ábra: Trajektóriák az eredeti állapottérben Rössler rendszerben

Figure 4

The variable x in time in case of the Rössler system



4. ábra: Az x változó alakulása az időben Rössler rendszerben

The lag time was 20τ in this case where τ is the sampling time $\tau = 0.05$. The lag time was chosen as the first minimum of the average mutual information function.

As *Figure 5* shows, the proposed indices (minimum eigenvalues and prediction errors) form a plot that correctly reflects the dimensionality of the original three dimensional system. Furthermore, the identified model gave excellent prediction performance (see *Figure 6*), the original and the predicted data take the same subspace. Because the model order is determined by finding the number of lagged outputs d_e indices form a table in this dimension. The predicted data depicted in *Figure 6* was given by a free run simulation of 5000 data. It can be seen that the trajectories are similar and the solution is good.

The second example in this paper is a four dimensional system published by *Yao* (2002). The system equations are

$$\ddot{x} = -(\alpha + y^2)x + y \tag{24}$$

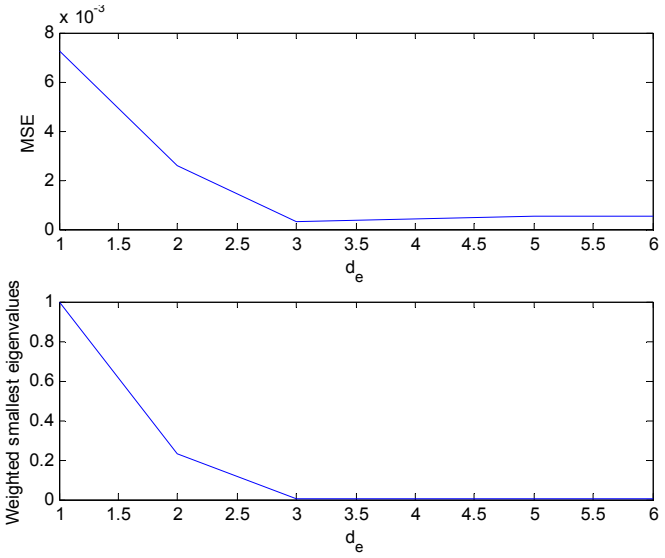
$$\ddot{y} = -(\beta + x^2)y + x \tag{25}$$

When $\alpha = 0.1$, $\beta = 0.101$ and initial conditions $x(0) = 0.1$, $\dot{x}(0) = 0.1$, $y(0) = -0.1$, $\dot{y}(0) = -0.1$, the 4 dimensional system is highly chaotic *Yao* (2002). The sampling rate was 0.05, and the lag time is 50τ by the average mutual information function.

As *Figure 7* shows the estimated embedding dimension is four, and the local dimension is three based on the weighted smallest eigenvalues (19). In *Figure 8* the first three dimensions of the original (solid line) and the predicted (dashed line) four dimensional data can be seen and the prediction performance is excellent, although the fourth dimension cannot be plotted.

Figure 5

Estimation of the d_e and d_l dimensions of the reconstruction space in case of a three dimensional chaotic system

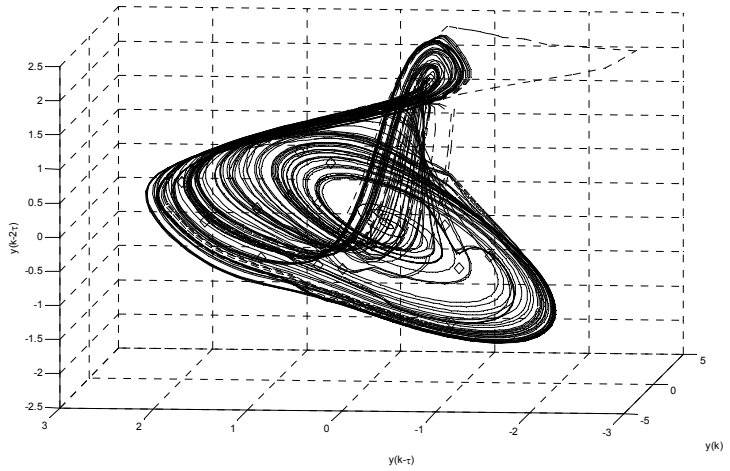


5. ábra: A d_e és d_l dimenziók becslése a rekonstrukciós tér alapján háromdimenziós kaotikus rendszerben

Súlyozott legkisebb sajátértékek(1)

Figure 6

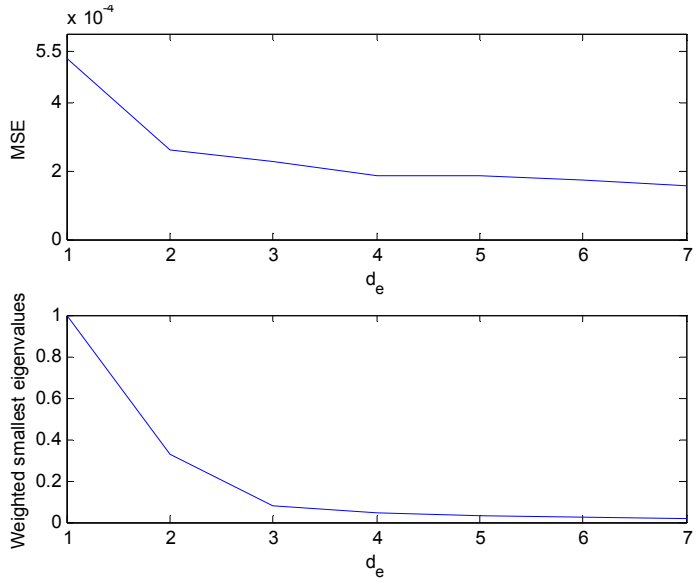
Prediction performance in case of a three dimensional chaotic system



6. ábra: Predikciós teljesítmény háromdimenziós kaotikus rendszerben

Figure 7

Estimation of the d_e and d_l dimensions of the reconstruction space in case of a four dimensional chaotic system

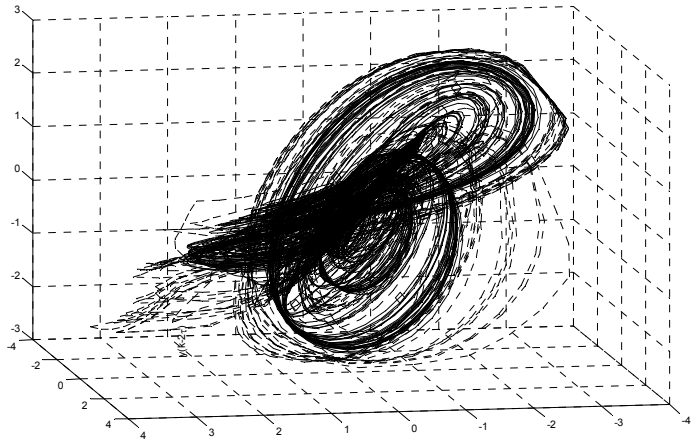


7. ábra: A d_e és d_l dimenziók becslése a rekonstrukciós tér alapján négydimenziós kaotikus rendszerben

Súlyozott legkisebb sajátértékek(1)

Figure 8

Prediction performance in case of a four dimensional chaotic system



8. ábra: Predikciós teljesítmény négydimenziós kaotikus rendszerben

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Corresponding author (*levelezési cím*):

János Abonyi
University of Veszprém, Department of Process Engineering
H-8201 Veszprém, P.O.Box 158
Veszprémi Egyetem, Folyamatmérnöki Tanszék
8201 Veszprém, Pf. 158.
Tel.: +36-88-624 209, fax: +36-88-624 171
E-mail: abonyij@fmt.vein.hu